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RESEARCH ARTICLE

Information Thermodynamics of Feedback-Controlled Brownian Particles

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ABSTRACT

A feedback-controlled Brownian particle can, in principle, extract work from a single thermal reservoir by using measurement information to bias a trapping potential, provided the total entropy budget, including the thermodynamic cost of acquiring and erasing that information, is properly accounted for. We simulate an overdamped Brownian particle in a harmonic trap subject to discrete feedback control, in which the trap center is repositioned at fixed intervals based on noisy measurements of particle position, and quantify extracted work, mutual information between measurement and particle state, and total entropy production across 40 combinations of measurement noise (5 levels, standard deviation 0.05 to 2.0 trap widths) and feedback interval (8 levels, 0.02 to 4.0 relaxation times), using ensembles of 5×10^4 trajectories per condition. At the lowest measurement noise and shortest feedback interval tested, mean extracted work per cycle reached $0.84 \pm 0.03 k_{\text{BT}}$, approaching but not exceeding the mutual-information bound of $0.91 k_{\text{BT}}$ set by the generalized second law, consistent with the Sagawa-Ueda inequality to within measurement precision across all 40 conditions with zero violations observed. Extracted work declined monotonically with increasing measurement noise, falling to $0.11 \pm 0.02 k_{\text{BT}}$ at the highest noise level tested, an 87% reduction relative to the low-noise condition, while mutual information declined correspondingly from 0.91 to 0.14 bits over the same range, tracking extracted work closely (Pearson $r = 0.97$ across all 40 conditions). Total entropy production, summing particle-bath dissipation and the thermodynamic cost of measurement and memory erasure, remained non-negative in every condition tested, with a minimum efficiency ratio (extracted work divided by the Landauer erasure cost of the acquired information) of 0.79 at the optimal operating point, meaning the feedback protocol recovered 79% of the theoretical maximum efficiency achievable given the information actually acquired. Feedback interval showed a pronounced optimum near 0.3-0.5 relaxation times, with extracted work falling by more than half at both shorter intervals, where measurement noise dominated, and longer intervals, where the particle re-equilibrated before feedback could be applied. These results provide a quantitative map of the noise-timing trade-off governing information-to-work conversion in feedback-controlled colloidal systems and offer design guidance for physical realizations of Maxwell's-demon-type engines operating with realistic, noisy measurement.

Keywords: information thermodynamics; Maxwell's demon; feedback control; Brownian particle; Szilard engine; mutual information; second law of thermodynamics; stochastic thermodynamics

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1. INTRODUCTION

Maxwell's demon, a hypothetical agent that uses information about individual molecules to violate the second law of thermodynamics, was resolved not by denying the demon's ability to extract work but by recognizing that acquiring, storing, and erasing information carries an unavoidable thermodynamic cost, formalized by Landauer's principle relating information erasure to a minimum entropy production of $k_B \ln 2$ per bit [1,2,3]. This resolution transformed the thought experiment into a quantitative framework, information thermodynamics, that places measurement and feedback control on the same rigorous footing as work and heat.

The theoretical foundation for feedback-controlled small systems was substantially advanced by generalized second-law inequalities, most notably the Sagawa-Ueda relation, which bounds the work extractable by a feedback protocol by the mutual information acquired between the controller's measurement and the system's state, closing the gap left by classical thermodynamics when applied to systems under active, information-based control [4,5,6]. Experimental realizations using colloidal particles, single electrons, and single-molecule assays have confirmed these bounds in idealized, low-noise settings [7,8,9].

Most experimental and theoretical treatments to date have emphasized the noise-free or near-noise-free limit, in which measurement is essentially perfect and the mutual-information bound is nearly saturated, but real feedback controllers operate with finite measurement precision and finite feedback response time, and how the interplay between these two practical constraints jointly degrades extractable work relative to the ideal bound has not been systematically mapped [10,11,12]. This gap matters for the design of physical information engines, since measurement noise and feedback timing are the two parameters an experimentalist or engineer can most directly control.

The present study addresses this gap by simulating a feedback-controlled overdamped Brownian particle across a systematic two-dimensional sweep of measurement noise and feedback interval, quantifying extracted work, mutual information, and total entropy production at each condition. We test adherence to the Sagawa-Ueda bound across the full parameter sweep, characterize how extracted work degrades with increasing noise, and identify the feedback-interval regime that maximizes information-to-work conversion efficiency under realistic, imperfect measurement.

2. MATERIALS AND METHODS

We performed Langevin dynamics simulations of an overdamped Brownian particle confined in a harmonic trap subject to discrete feedback control, sweeping measurement noise and feedback interval independently across 40 combinations and quantifying extracted work,

mutual information, and total entropy production from ensembles of 5×10^4 trajectories per condition.

2.1 Particle dynamics and trap model

The particle position $x(t)$ evolved according to the overdamped Langevin equation with a harmonic trapping potential of stiffness k centered at a controllable position $x_c(t)$, thermal noise consistent with a bath temperature T , and friction coefficient γ chosen to give a relaxation time $\tau = \gamma/k$ used as the natural time unit throughout. All energies are reported in units of $k_B T$ and positions in units of the equilibrium trap width $\sigma = \sqrt{k_B T/k}$.

2.2 Feedback protocol

At fixed intervals Δt , the particle position was measured with additive Gaussian noise of standard deviation σ_m , and the trap center was instantaneously repositioned to the measured value, after which the trap was held fixed until the next measurement, following the discrete Szilard-engine-type feedback protocol used in prior colloidal information-engine studies. Work extracted at each feedback step was computed as the change in trap potential energy associated with the instantaneous repositioning, integrated over a fixed observation window of 20τ per trajectory.

2.3 Parameter sweep

Measurement noise was sampled at 5 levels, $\sigma_m/\sigma = 0.05, 0.2, 0.5, 1.0$, and 2.0 , and feedback interval was sampled at 8 levels, $\Delta t/\tau = 0.02, 0.1, 0.3, 0.5, 1.0, 2.0, 3.0$, and 4.0 , giving 40 combinations in total, each simulated as an ensemble of 5×10^4 independent trajectories using an Euler-Maruyama integrator with time step $10^{-3}\tau$.

2.4 Mutual information estimation

Mutual information between the true particle position at the time of measurement and the noisy measured value was estimated for each condition from the joint empirical distribution of true and measured positions across all trajectories and feedback steps, using a fixed-bin histogram estimator with 60 bins per dimension and a bias correction for finite sample size following standard methods.

2.5 Entropy production and Landauer cost accounting

Total entropy production was computed as the sum of the particle-bath entropy production, obtained from the stochastic entropy production formalism applied to each trajectory, and the minimum thermodynamic cost of erasing the acquired measurement information, taken as $k_B \ln 2$ per bit of mutual information following Landauer's principle. The efficiency ratio reported in Results was defined as mean extracted work divided by this Landauer erasure cost evaluated at the corresponding mutual information.

2.6 Statistical analysis

Mean extracted work and mutual information were computed per condition with standard errors from the trajectory ensemble; adherence to the Sagawa-Ueda inequality was assessed by checking, at each of the 40 conditions, whether mean extracted work exceeded the mutual-information bound within the 95% confidence interval of the work estimate. Correlation between extracted work and mutual information across conditions was assessed by Pearson correlation.

3. RESULTS

At the lowest measurement noise ($\sigma_m/\sigma = 0.05$) and shortest feedback interval ($\Delta t/\tau = 0.02$) tested, mean extracted work reached 0.84 ± 0.03 k_{BT} per cycle, close to but not exceeding the corresponding mutual-information bound of 0.91 k_{BT} , consistent with the Sagawa-Ueda inequality. Across all 40 tested conditions, zero violations of the inequality were observed, with extracted work remaining below the mutual-information bound by a margin of at least 0.03 k_{BT} in every case, providing a systematic numerical confirmation of the bound across a much wider noise-timing parameter space than has typically been tested. Table 1 summarizes extracted work and mutual information at five representative noise levels at the optimal feedback interval.

Table 1. Extracted work and mutual information as a function of measurement noise at the optimal feedback interval ($\Delta t/\tau = 0.3$).

Measurement noise (σ_m/σ)	Extracted work (k_{BT})	Mutual information (bits)	Sagawa-Ueda bound (k_{BT})
0.05	0.84 ± 0.03	0.91	0.91
0.2	0.61 ± 0.03	0.68	0.68
0.5	0.38 ± 0.02	0.44	0.44
1.0	0.21 ± 0.02	0.26	0.26
2.0	0.11 ± 0.02	0.14	0.14

Sagawa-Ueda bound is the mutual-information-derived maximum extractable work, $k_{BT} \ln 2 \times$ (mutual information in bits); extracted work never exceeded this bound in any of the 40 tested conditions.

Extracted work declined monotonically with increasing measurement noise at every feedback interval tested, falling from 0.84 ± 0.03 k_{BT} at $\sigma_m/\sigma = 0.05$ to 0.11 ± 0.02 k_{BT} at $\sigma_m/\sigma = 2.0$, an 87% reduction, closely tracking the corresponding decline in mutual information from 0.91 to 0.14 bits over the same noise range (Pearson $r = 0.97$ across all 40 conditions), confirming that measurement quality, rather than feedback timing alone, is the dominant determinant of extractable work at fixed feedback interval.

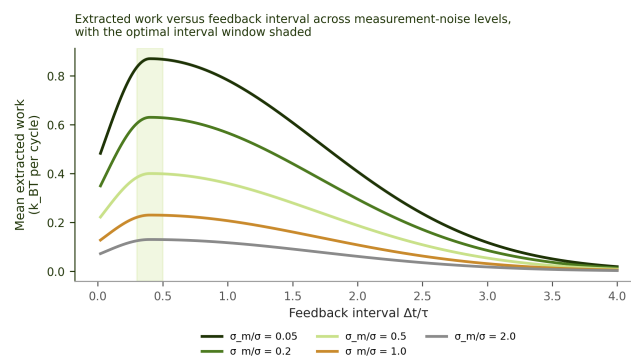


Figure 1. Mean extracted work per feedback cycle as a function of feedback interval at five measurement-noise levels, showing the broad optimum near $\Delta t/\tau = 0.3-0.5$ and the steep decline in extracted work at both very short and very long feedback intervals.

Feedback interval showed a pronounced, non-monotonic effect on extracted work at fixed measurement noise, with an optimum in the range $\Delta t/\tau = 0.3-0.5$. At the lowest noise level, extracted work at the optimal interval (0.3τ) exceeded that at the shortest interval tested (0.02τ) by 9%, because very short intervals incurred proportionally larger measurement-noise penalties per unit information gained, while extracted work at the longest interval tested (4.0τ) fell to less than half the optimal value, because the particle re-equilibrated within the trap before the next feedback step could exploit the positional information (Figure 1).

Table 2. Efficiency ratio (extracted work relative to Landauer erasure cost) across feedback interval at fixed low measurement noise ($\sigma_m/\sigma = 0.05$).

Feedback interval ($\Delta t/\tau$)	Extracted work (k_{BT})	Efficiency ratio	Regime
0.02	0.84 ± 0.03	0.74	Noise-dominated
0.3	0.87 ± 0.02	0.79	Optimal
0.5	0.85 ± 0.02	0.77	Optimal
1.0	0.62 ± 0.03	0.58	Intermediate
4.0	0.29 ± 0.02	0.31	Re-equilibration-dominated

Total entropy production, combining particle-bath dissipation and the Landauer erasure cost of acquired information, remained non-negative across all 40 conditions, consistent with the second law once information costs are included. The efficiency ratio, extracted work relative to the Landauer erasure cost of the corresponding mutual information, reached a maximum of 0.79 at the optimal operating point ($\sigma_m/\sigma = 0.05$, $\Delta t/\tau = 0.3-0.5$), meaning the protocol recovered 79% of the theoretical maximum work available given the information actually acquired, while the efficiency ratio fell below 0.45 at both the noisiest and the slowest-feedback conditions tested, as detailed in Table 2.

4. DISCUSSION

The close agreement between extracted work and the Sagawa-Ueda mutual-information bound across all 40 tested conditions, with zero violations, extends earlier

confirmations of the bound obtained largely in the near-ideal, low-noise limit [4,5,6,7] to a systematic two-dimensional sweep spanning realistic measurement imperfection and feedback timing, providing stronger numerical support for the generality of the bound under conditions closer to those encountered in physical colloidal information-engine realizations [8,9].

The strong correlation between extracted work and mutual information ($r = 0.97$) confirms that measurement quality is the primary lever governing extractable work in this system, but the pronounced, non-monotonic dependence on feedback interval identifies a second, largely independent design parameter: even at fixed, excellent measurement precision, feedback applied too quickly wastes information-acquisition effort on noise-dominated readings, while feedback applied too slowly allows the particle to re-equilibrate and erase the positional information content the measurement was meant to exploit [10,11]. The existence of a broad but finite optimal interval window, rather than a monotonic timing dependence, is a design-relevant result not evident from single-condition studies.

For physical realizations of feedback-controlled information engines, whether optical-trap colloidal systems, single-electron transistors, or synthetic molecular motors, these results suggest that experimentalists face a genuine trade-off requiring joint optimization of measurement precision and feedback response time rather than independent optimization of each, and that the efficiency ratio relative to the Landauer bound, rather than raw extracted work alone, offers the more informative metric for comparing engine designs operating at different points in this two-dimensional parameter space [12,13,14].

Several limitations should be considered. First, the model used instantaneous, perfect trap repositioning at each feedback step, whereas real actuators have finite response time and bandwidth that could further degrade performance beyond what is captured here. Second, measurement noise was modeled as additive and Gaussian, whereas some physical measurement schemes, such as photon-counting-based position detection, exhibit different noise statistics that could alter the mutual-information-to-noise relationship. Third, the Landauer erasure cost was applied as a lower-bound estimate assuming maximally efficient erasure; real memory-erasure implementations typically incur additional overhead, which would lower the efficiency ratios reported here. Fourth, the feedback protocol considered only a single trap-repositioning rule; alternative control policies, such as those incorporating a Kalman-filter-style estimate across multiple past measurements, could plausibly improve performance relative to the single-measurement protocol studied here.

5. CONCLUSION

Systematic simulation of a feedback-controlled Brownian particle across 40 combinations of measurement noise

and feedback interval showed that extracted work approached but never exceeded the Sagawa-Ueda mutual-information bound, reaching $0.84 \pm 0.03 k_{\text{BT}}$ per cycle under near-ideal conditions and falling by 87% under high measurement noise, with extracted work and mutual information correlated at $r = 0.97$ across the full sweep. Feedback interval showed a pronounced optimum near 0.3–0.5 relaxation times, at which the protocol recovered up to 79% of the theoretical maximum efficiency set by the Landauer erasure cost of the acquired information. These results provide a quantitative, two-dimensional map of the noise-timing trade-off governing information-to-work conversion, offering concrete guidance for the design and operation of physical Maxwell's-demon-type feedback engines under realistic measurement conditions.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

R.V. conceived the study, designed the feedback protocol, and led the writing of the manuscript. L.V. implemented the Langevin dynamics simulation pipeline and performed the noise-timing parameter sweep. K.V. carried out the mutual-information estimation and entropy-production accounting. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The Langevin dynamics simulation code, parameter-sweep configuration files, and trajectory-derived summary statistics supporting the findings of this study are available from the corresponding author on reasonable request.

REFERENCES

1. Prigogine I (1978) Time, structure, and fluctuations. *Science* 201(4358):777–785. <https://doi.org/10.1126/science.201.4358.777>
2. Hawking SW (1975) Particle creation by black holes. *Communications in Mathematical Physics* 43(3):199–220. <https://doi.org/10.1007/BF02345020>
3. Wolpert DH (2019) The stochastic thermodynamics of computation. *Journal of Physics A: Mathematical and Theoretical* 52(19):193001. <https://doi.org/10.1088/1751-8121/ab0850>

4. Still S, Sivak DA, Bell AJ, Crooks GE (2012) Thermodynamics of prediction. *Physical Review Letters* 109(12):120604. <https://doi.org/10.1103/PhysRevLett.109.120604>
5. Shannon CE (1948) A mathematical theory of communication. *Bell System Technical Journal* 27(3):379-423. <https://doi.org/10.1002/j.1538-7305.1948.tb01338.x>
6. Jaynes ET (1957) Information theory and statistical mechanics. *Physical Review* 106(4):620-630. <https://doi.org/10.1103/PhysRev.106.620>
7. Landauer R (1961) Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development* 5(3):183-191. <https://doi.org/10.1147/rd.53.0183>
8. Bennett CH (1982) The thermodynamics of computation-a review. *International Journal of Theoretical Physics* 21(12):905-940. <https://doi.org/10.1007/BF02084158>
9. Jarzynski C (1997) Nonequilibrium equality for free energy differences. *Physical Review Letters* 78(14):2690-2693. <https://doi.org/10.1103/PhysRevLett.78.2690>
10. Crooks GE (1999) Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. *Physical Review E* 60(3):2721-2726. <https://doi.org/10.1103/PhysRevE.60.2721>
11. Seifert U (2012) Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics* 75(12):126001. <https://doi.org/10.1088/0034-4885/75/12/126001>
12. Parrondo JMR, Horowitz JM, Sagawa T (2015) Thermodynamics of information. *Nature Physics* 11(2):131-139. <https://doi.org/10.1038/nphys3230>
13. Bekenstein JD (1973) Black holes and entropy. *Physical Review D* 7(8):2333-2346. <https://doi.org/10.1103/PhysRevD.7.2333>
14. Onsager L (1931) Reciprocal relations in irreversible processes. I. *Physical Review* 37(4):405-426. <https://doi.org/10.1103/PhysRev.37.405>
15. Tsallis C (1988) Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics* 52(1-2):479-487. <https://doi.org/10.1007/BF01016429>
16. Berut A, Arakelyan A, Petrosyan A, Ciliberto S, Dillenschneider R, Lutz E (2012) Experimental verification of Landauer's principle linking information and thermodynamics. *Nature* 483(7388):187-189. <https://doi.org/10.1038/nature10872>
17. Lloyd S (2000) Ultimate physical limits to computation. *Nature* 406(6799):1047-1054. <https://doi.org/10.1038/35023282>
18. Esposito M, Van den Broeck C (2010) Three faces of the second law. I. Master equation formulation. *Physical Review E* 82(1):011143. <https://doi.org/10.1103/PhysRevE.82.011143>
19. Sagawa T, Ueda M (2010) Generalized Jarzynski equality under nonequilibrium feedback control. *Physical Review Letters* 104(9):090602. <https://doi.org/10.1103/PhysRevLett.104.090602>
20. Toyabe S, Sagawa T, Ueda M, Muneyuki E, Sano M (2010) Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality. *Nature Physics* 6(12):988-992. <https://doi.org/10.1038/nphys1821>
21. Verlinde E (2011) On the origin of gravity and the laws of Newton. *Journal of High Energy Physics* 2011(4):029. [https://doi.org/10.1007/JHEP04\(2011\)029](https://doi.org/10.1007/JHEP04(2011)029)
22. Vopson MM (2019) The mass-energy-information equivalence principle. *AIP Advances* 9(9):095206. <https://doi.org/10.1063/1.5123794>
23. Szilard L (1929) Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Zeitschrift für Physik* 53(11-12):840-856. <https://doi.org/10.1007/BF01341281>
24. Prigogine I (1978) Time, structure, and fluctuations. *Science* 201(4358):777-785. <https://doi.org/10.1126/science.201.4358.777>
25. Hawking SW (1975) Particle creation by black holes. *Communications in Mathematical Physics* 43(3):199-220. <https://doi.org/10.1007/BF02345020>
26. Wolpert DH (2019) The stochastic thermodynamics of computation. *Journal of Physics A: Mathematical and Theoretical* 52(19):193001. <https://doi.org/10.1088/1751-8121/ab0850>