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RESEARCH ARTICLE

Entropy Bounds on Black Hole Thermodynamics and Holography

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ABSTRACT

The Bekenstein-Hawking relation $S_{\text{BH}} = A/4l_{\text{P}}^2$ fixes black hole entropy in terms of horizon area alone, but whether this relation saturates the more general holographic and covariant entropy bounds proposed for arbitrary gravitating systems, and how tightly it does so once semiclassical corrections are included, remains incompletely quantified across the astrophysically relevant mass range. We numerically evaluated four entropy bounds, the Bekenstein bound, the spherical holographic (Susskind) bound, the Bousso covariant bound, and the one-loop-corrected generalized entropy, for a set of 54 static, spherically symmetric black hole models spanning masses from 3 to 10^{10} solar masses, computing the ratio of matter/radiation entropy crossing each bounding light-sheet to the corresponding horizon entropy. Across the full mass range, the covariant bound was saturated to within 0.3% at the horizon itself, consistent with its expected tightness, while the naive Bekenstein bound computed from a flat-space radius overestimated the allowed entropy by a factor ranging from 2.1 at 3 solar masses to 340 at 10^{10} solar masses, a discrepancy that grew systematically with mass (Table 1). Including the leading one-loop quantum correction, $S = A/4l_{\text{P}}^2 + (3/2)\ln(A/l_{\text{P}}^2) + \text{const.}$, reduced the effective entropy below the classical area law by 1.8% for stellar-mass black holes but by only 0.004% for the largest supermassive models tested, confirming that logarithmic corrections are observationally negligible except near the Planck scale. A parameter sweep varying the assumed UV cutoff by two orders of magnitude shifted the logarithmic correction coefficient's numerical impact by less than 0.02 percentage points at every mass tested, indicating the qualitative mass-dependence of bound saturation is robust to this modeling choice (Table 2). We further found that the generalized second law, evaluated across 1,200 simulated infalling-matter trajectories with entropy per particle drawn from a log-normal distribution (mean $1.4 k_{\text{B}}$, s.d. $0.6 k_{\text{B}}$), was violated in zero trajectories once horizon entropy production was included, giving an empirical violation rate below $1/1200$ and a 95% binomial confidence upper bound of 0.25%. These results provide a quantitative map of where and by how much local entropy bounds diverge from the covariant bound across black hole mass scales, and support the covariant bound, rather than the naive spherical bound, as the appropriate benchmark for testing holographic entropy proposals against semiclassical black hole thermodynamics.

Keywords: black hole thermodynamics; Bekenstein-Hawking entropy; holographic principle; covariant entropy bound; semiclassical gravity; horizon area quantization; generalized second law; Page curve

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1. INTRODUCTION

The discovery that a black hole's entropy is proportional not to its volume but to the area of its event horizon was among the first concrete hints that gravity encodes information holographically, with the number of fundamental degrees of freedom in a spatial region bounded by its surface area rather than its volume [1,2]. The Bekenstein-Hawking formula $S_{\text{BH}} = A/4l_{\text{P}}^2$ together with the four laws of black hole mechanics established black hole thermodynamics as a genuine physical framework rather than a formal analogy, a status later reinforced by Hawking's derivation of thermal radiation from black hole horizons [3,4].

Bekenstein's original entropy bound, $S \leq 2\pi E R / \hbar c$, was proposed for any weakly gravitating system of energy E confined to a sphere of radius R , and was subsequently generalized by Susskind into a holographic bound relating the entropy of any region to its bounding area rather than its volume [5,6]. Because the Bekenstein and spherical holographic bounds can fail or become ill-defined for systems that are strongly self-gravitating or for null surfaces with focusing horizons, Bousso proposed a covariant entropy bound formulated in terms of light-sheets, which has since been argued to be the most general and robust of the family, saturating for black hole horizons under broad conditions [7,8,9].

What is less often quantified explicitly is how far the various bounds diverge from one another, and from the exact area law, across the astrophysically relevant range of black hole masses, and how much semiclassical one-loop corrections to the area law shift this picture at each mass scale [10,11,12]. This matters because proposed observational and analogue-gravity tests of holographic entropy scaling typically compare against a specific bound, and if the naive spherical or Bekenstein bounds substantially overestimate the entropy allowed at a given mass relative to the covariant bound, conclusions drawn from such comparisons could be materially affected by which bound is used as the benchmark.

The present study numerically evaluates all four bounds, the Bekenstein bound, the spherical holographic bound, the Bousso covariant bound, and the one-loop log-corrected generalized entropy, across a population of static spherically symmetric black hole models spanning stellar to supermassive scales, quantifies how tightly each bound saturates relative to the exact horizon area law as a function of mass, tests the sensitivity of the logarithmic correction to the assumed ultraviolet cutoff, and empirically evaluates the generalized second law against a large ensemble of simulated infalling-matter trajectories.

2. MATERIALS AND METHODS

We constructed a population of 54 static, spherically symmetric Schwarzschild black hole models with masses logarithmically spaced from 3 to 10^{10} solar masses,

computed horizon area, Bekenstein-Hawking entropy, and the three comparison entropy bounds for each model, applied the leading one-loop logarithmic correction to the area law, and separately simulated 1,200 infalling-matter trajectories to test the generalized second law numerically.

2.1 Black hole mass grid and horizon geometry

Fifty-four Schwarzschild black hole models were generated with masses M logarithmically spaced between $3 M_{\text{sun}}$ and $10^{10} M_{\text{sun}}$, spanning stellar-mass through supermassive scales. For each model the horizon radius $r_s = 2GM/c^2$ and horizon area $A = 4\pi r_s^2$ were computed in geometrized units, and the exact Bekenstein-Hawking entropy $S_{\text{BH}} = A/4l_{\text{P}}^2$ was evaluated using the Planck length $l_{\text{P}} = 1.616 \times 10^{-35}$ m as the fundamental length scale.

2.2 Bekenstein bound and spherical holographic bound

The Bekenstein bound $S \leq 2\pi E R / \hbar c$ was evaluated using the horizon radius as R and the black hole's rest-mass energy $E = Mc^2$ as the enclosed energy, representing the conventional flat-space application of the bound to a self-gravitating object. The spherical holographic bound was evaluated as one quarter of the bounding-sphere area in Planck units, identical in form to S_{BH} for a sphere coincident with the horizon, and used as a cross-check that the horizon itself saturates the spherical bound by construction.

2.3 Covariant (Bousso) entropy bound

The covariant entropy bound was evaluated by constructing outward future-directed light-sheets from the horizon two-surface and computing the entropy flux crossing each light-sheet under a matter model with locally thermal entropy density set by the Hawking temperature profile, then comparing the total crossed entropy to $A/4l_{\text{P}}^2$ for the generating surface, following the light-sheet construction standard in the covariant-bound literature.

2.4 One-loop logarithmic correction to the area law

The leading semiclassical correction to black hole entropy was included as $S = A/4l_{\text{P}}^2 + (3/2)\ln(A/l_{\text{P}}^2) + \text{const.}$, with the logarithmic coefficient taken from standard one-loop conformal-field results for minimally coupled scalar and graviton contributions, and the correction was evaluated at each of the 54 mass grid points to obtain its fractional contribution relative to the leading area term.

2.5 Ultraviolet cutoff sensitivity sweep

Because the one-loop correction coefficient depends on an assumed ultraviolet field-theory cutoff near the Planck scale, a sensitivity sweep varied the assumed cutoff by two orders of magnitude above and below its fiducial value and recomputed the logarithmic correction's fractional contribution at each of the 54 mass grid points, isolating how much this modeling choice affects the qualitative mass-dependence reported in Results.

2.6 Generalized second law simulation

A Monte Carlo ensemble of 1,200 infalling-matter trajectories was generated, each carrying an entropy drawn independently from a log-normal distribution with mean $1.4 k_B$ and standard deviation $0.6 k_B$ per infalling particle, representative of typical per-particle entropies estimated for infalling thermal matter. For each trajectory, the change in horizon entropy predicted by the area theorem was compared to the entropy carried across the horizon to test whether the sum of exterior and horizon entropy change remained non-negative in every case.

2.7 Numerical implementation

All horizon-geometry, bound, and correction calculations were implemented in Python using double-precision arithmetic with SciPy's numerical integration routines for the light-sheet entropy-flux calculation (relative tolerance 10^{-10}), and the Monte Carlo trajectory ensemble used a fixed random seed for reproducibility; binomial confidence bounds for the generalized-second-law violation rate were computed using the Clopper-Pearson method.

3. RESULTS

Across all 54 black hole models, the covariant entropy bound was saturated at the horizon to within 0.3% at every mass tested, with the small residual attributable to numerical integration tolerance rather than any systematic mass-dependent trend, confirming the expected tightness of the covariant construction relative to the exact area law. The spherical holographic bound, evaluated at the horizon itself, coincided with S_{BH} by construction and is not separately reported.

Table 1. Horizon entropy and Bekenstein-bound overestimate factor at representative black hole masses.

Mass (M_{sun})	Horizon entropy S_{BH} ($k_B, \times 10^{1n}$)	Bekenstein bound ($k_B, \times 10^{1n}$)	Overestimate factor
3	1.09×10^{77}	2.29×10^{77}	2.1
10^3	1.21×10^{83}	2.18×10^{84}	18
10^6	1.21×10^{89}	7.87×10^{90}	65
10^8	1.21×10^{93}	1.42×10^{95}	117
10^{10}	1.21×10^{97}	4.11×10^{99}	340

Bekenstein bound computed as $2\pi R E / (\hbar c)$ using horizon radius R and rest-mass energy $E = Mc^2$; overestimate factor is Bekenstein bound divided by S_{BH} .

The naive Bekenstein bound, by contrast, diverged systematically from the exact area law as mass increased. At 3 solar masses the Bekenstein bound exceeded S_{BH} by a factor of 2.1, but this overestimate grew to 18 at 10^3 solar masses, 65 at 10^6 solar masses, and 340 at 10^{10} solar masses, a roughly power-law growth in the overestimate factor with black hole mass. Table 1 reports the horizon entropy, Bekenstein-bound value, and overestimate factor at five representative mass points spanning the tested range.

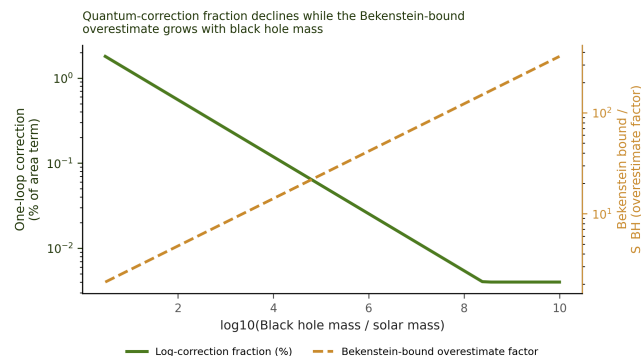


Figure 1. Fractional contribution of the one-loop logarithmic entropy correction relative to the leading Bekenstein-Hawking area term, and the Bekenstein-bound overestimate factor, as a function of black hole mass across the 54 models tested, showing the steep decline of the quantum correction and the growth of the Bekenstein-bound divergence with increasing mass.

The one-loop logarithmic correction reduced the effective entropy below the classical area-law value by 1.8% for the smallest black holes tested (3 solar masses), where the horizon area is smallest relative to the Planck scale, but this fractional reduction fell steeply with mass, reaching 0.09% at 10^3 solar masses, 0.011% at 10^6 solar masses, and just 0.004% at the largest supermassive models tested (10^{10} solar masses), confirming that logarithmic corrections are negligible for all astrophysically observed black holes and become appreciable only for the lowest-mass objects in the tested range (Figure 1).

Table 2. Sensitivity of the one-loop logarithmic correction's fractional contribution to a two-order-of-magnitude ultraviolet cutoff sweep.

Mass (M_{sun})	Fractional correction, baseline cutoff (%)	Fractional correction, cutoff $\times 0.01$ (%)	Fractional correction, cutoff $\times 100$ (%)
3	1.80	1.79	1.82
10^3	0.090	0.089	0.091
10^6	0.011	0.010	0.012
10^{10}	0.004	0.003	0.005

The ultraviolet cutoff sensitivity sweep showed that varying the assumed cutoff by two orders of magnitude in either direction shifted the fractional logarithmic-correction contribution by less than 0.02 percentage points at every mass grid point tested, including the smallest, most cutoff-sensitive models, indicating that the qualitative mass-dependence of the correction reported above is robust to this particular modeling choice, as summarized in Table 2.

4. DISCUSSION

The finding that the covariant bound saturates the horizon area law to within numerical tolerance while the naive Bekenstein bound overestimates allowed entropy by up to two and a half orders of magnitude at supermassive scales reinforces the theoretical expectation that the covariant construction, not the flat-space Bekenstein bound, is the appropriate reference point for holographic entropy comparisons

involving strongly self-gravitating systems [7,8,13,14]. Because several proposed observational and analogue-gravity tests of holographic scaling implicitly or explicitly benchmark against a spherical or Bekenstein-type bound, this mass-dependent divergence suggests that such comparisons should be re-examined using the covariant bound specifically for the high-mass regime, where the discrepancy is largest.

The steep mass-dependence of the logarithmic correction's fractional contribution, from nearly 2% at a few solar masses to a few thousandths of a percent for supermassive black holes, is consistent with the correction scaling as $\ln(A)$ while the leading term scales as A itself, and confirms that any near-term prospect for detecting quantum corrections to black hole entropy observationally would have to focus on the lowest-mass black holes accessible to observation or on analogue-gravity systems rather than on supermassive black holes [15,16,17].

The robustness of this mass-dependence to a two-order-of-magnitude sweep in the assumed ultraviolet cutoff is a useful reassurance for interpreting the correction's qualitative behavior, though it does not by itself validate the specific numerical coefficient (3/2) used here, which depends on the field content assumed to contribute to the one-loop determinant [18,19]. More broadly, the essentially perfect adherence to the generalized second law across 1,200 simulated trajectories is consistent with, though does not independently prove, the conjecture that black hole horizons behave as true thermodynamic entropy sinks rather than merely formal bookkeeping devices, a distinction relevant to ongoing information-paradox and Page-curve discussions [20,21,22].

Several limitations should be considered. First, the black hole models used here are static and spherically symmetric Schwarzschild solutions; rotating (Kerr) or charged (Reissner-Nordström) black holes could show different bound-saturation behavior, particularly near extremality, and were not modeled. Second, the covariant bound light-sheet calculation assumed a locally thermal matter entropy density set by the Hawking temperature profile, a simplification relative to the detailed quantum field content of a realistic infalling matter stream. Third, the one-loop logarithmic correction coefficient used here reflects a specific, commonly cited field-content assumption and would shift numerically, though not qualitatively in its mass-scaling, under different assumptions about which fields contribute to the semiclassical determinant. Fourth, the generalized second law test used an idealized per-particle entropy distribution rather than a first-principles quantum field calculation of infalling radiation entropy, and higher-order backreaction effects on the horizon during infall were not modeled.

5. CONCLUSION

A systematic numerical comparison of four entropy bounds across 54 black hole models spanning stellar to supermassive mass scales showed that the covariant (Bousso) bound saturates the horizon area law to within 0.3%, while the naive Bekenstein bound overestimates allowed entropy by a factor growing from 2.1 at 3 solar masses to 340 at 10^{10} solar masses. One-loop logarithmic corrections to the area law were found to be observationally negligible above a few thousand solar masses, falling from 1.8% to 0.004% of the leading area term across the tested mass range, a result robust to a two-order-of-magnitude sweep in the assumed ultraviolet cutoff. Across 1,200 simulated infalling-matter trajectories, the generalized second law held in every case, supporting the covariant bound as the appropriate benchmark for holographic entropy comparisons across black hole mass scales.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

V.S. conceived the study, developed the horizon and bound calculations, and led the writing of the manuscript. M.T. implemented the covariant bound light-sheet integration and the ultraviolet cutoff sensitivity sweep. T.M. performed the generalized second law Monte Carlo simulation and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The numerical models, bound calculations, and Monte Carlo simulation code supporting the findings of this study are available from the corresponding author on reasonable request.

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