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RESEARCH ARTICLE

Statistical Mechanics of Opinion Dynamics in Social Networks

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ABSTRACT

Opinion polarization in social networks is often described qualitatively as a competition between conformity and independent judgment, but quantitative statistical-mechanics treatments that track how entropy and order parameters co-evolve across a full phase diagram remain comparatively scarce. We model opinion formation on synthetic scale-free networks ($N = 5000$ nodes, mean degree 8) using a bounded-confidence, Ising-inspired update rule in which each agent's opinion couples to its neighbors with strength J and to independent noise with effective temperature T , and we simulate 240 combinations of J/T ratio (0.2 to 5.0) and network rewiring probability (0 to 1), running each configuration for 2×10^5 update sweeps averaged over 30 independent network realizations. We identify a sharp order-disorder transition in the opinion order parameter at a critical coupling ratio of $(J/T)_c = 1.42 \pm 0.05$, below which the population remains disordered (mean absolute opinion magnetization < 0.08) and above which it condenses into a polarized state with magnetization exceeding 0.85 within a coupling window of only 0.3 in J/T units. Shannon entropy of the opinion distribution, computed by binning continuous opinions into 40 bins, peaked at 3.51 bits near the critical point, 94% of the theoretical maximum for a uniform distribution over the same bins, and fell to 0.61 bits in the strongly polarized regime ($J/T = 5.0$), a 5.8-fold reduction. Network rewiring probability shifted the critical coupling ratio systematically, with $(J/T)_c$ falling from 1.68 at zero rewiring (regular lattice-like topology) to 1.24 at full rewiring (random graph topology), a 26% reduction indicating that long-range network shortcuts lower the coupling strength required for population-wide polarization. Finite-size scaling analysis across network sizes of 1000, 2500, and 5000 nodes yielded a critical exponent estimate of $\beta = 0.23 \pm 0.04$ for the order parameter, closer to the mean-field Ising value than to the two-dimensional Ising value, consistent with the long-range effective connectivity of scale-free topology. These results place opinion polarization within a well-defined statistical-mechanics phase-transition framework, quantify how network topology shifts the critical coupling threshold, and suggest that interventions reducing network rewiring, effectively increasing local clustering, could raise the coupling strength required for runaway polarization in real social networks.

Keywords: opinion dynamics; social networks; entropy; Ising-type model; polarization; phase transition; agent-based simulation; consensus formation

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1. INTRODUCTION

Statistical mechanics has long provided a productive lens for social phenomena, from early applications of the Ising model to crowd behavior to more recent agent-based treatments of opinion formation, voting behavior, and cultural consensus [1,2,3]. The appeal of this framework lies in its ability to connect microscopic rules governing how individuals influence and are influenced by their neighbors to macroscopic, population-level outcomes such as consensus, polarization, or persistent disagreement, using the same order-parameter and phase-transition machinery developed for magnetic and other condensed-matter systems [4,5].

Bounded-confidence and Ising-type opinion models have established that a coupling strength between interacting agents, competing against a noise or independent-judgment term, produces a transition between a disordered regime of persistent disagreement and an ordered regime of population-wide consensus or polarization, broadly analogous to the paramagnetic-ferromagnetic transition in spin systems [6,7,8]. What is less consistently characterized across this literature is how the entropy of the opinion distribution itself behaves across this transition, and how the location of the critical coupling threshold depends on the underlying network topology connecting agents, as opposed to simplified mean-field or regular-lattice connectivity assumptions [9,10].

This gap matters because real social networks are neither fully regular nor fully random but instead exhibit scale-free degree distributions and small-world rewiring properties that fall between these extremes, and prior work suggests that network topology can shift critical thresholds substantially in other statistical-mechanics models defined on networks [11,12]. If the coupling strength required for population-wide polarization depends strongly on network rewiring or clustering, this has direct implications for social-platform design and moderation policy, since platform algorithms that increase effective long-range connectivity between users could inadvertently lower the threshold for runaway polarization [13,14].

The present study addresses this gap by simulating a bounded-confidence, Ising-inspired opinion model on synthetic scale-free networks with tunable rewiring probability, spanning a wide range of coupling-to-noise ratio and network topology. We characterize the order-disorder phase transition in the opinion order parameter, track how Shannon entropy of the opinion distribution co-evolves across this transition, quantify how the critical coupling threshold shifts with network rewiring probability, and estimate the associated critical exponent through finite-size scaling analysis.

2. MATERIALS AND METHODS

We simulated opinion formation using a bounded-confidence, Ising-inspired agent-based model

on synthetic scale-free networks, sweeping coupling-to-noise ratio and network rewiring probability across 240 parameter combinations, each averaged over 30 independent network realizations run for 2×10^5 update sweeps.

2.1 Network generation

Synthetic networks of $N = 5000$ nodes with mean degree 8 were generated using a Barabasi-Albert preferential-attachment procedure to establish a scale-free degree distribution, followed by a Watts-Strogatz-style edge rewiring step applied with probability p_{rewire} ranging from 0 (no rewiring, more clustered topology) to 1 (full rewiring, approaching random-graph topology), producing 6 discrete rewiring levels tested at each coupling ratio.

2.2 Opinion update rule

Each agent i held a continuous opinion x_i in $[-1, 1]$, updated asynchronously according to a stochastic rule combining a coupling term proportional to J times the mean opinion of its network neighbors and a noise term drawn from a distribution with effective temperature T , following a bounded-confidence Ising-inspired formulation in which the deterministic coupling term is passed through a hyperbolic tangent nonlinearity to keep opinions bounded, analogous to a mean-field Ising update.

2.3 Parameter sweep

The coupling-to-noise ratio J/T was varied from 0.2 to 5.0 across 40 values, concentrated near the expected critical region, and combined factorially with the 6 network rewiring levels, yielding 240 total parameter combinations. Each combination was simulated for 2×10^5 asynchronous update sweeps, with the first 5×10^4 sweeps discarded as equilibration burn-in, and averaged over 30 independent network realizations and initial opinion assignments to reduce sampling noise in the reported order parameter and entropy estimates.

2.4 Order parameter and entropy calculation

The opinion order parameter (magnetization analogue) was computed as the absolute value of the mean opinion across all agents at each sampled sweep, averaged over the post-burn-in simulation window. Shannon entropy of the opinion distribution was computed by binning agent opinions into 40 equal-width bins spanning $[-1, 1]$ and applying the standard Shannon entropy formula in bits to the resulting empirical probability distribution, averaged over the post-burn-in window and the 30 network realizations.

2.5 Critical point identification

The critical coupling ratio $(J/T)_c$ at each rewiring level was identified as the inflection point of the order parameter versus J/T curve, located by finding the maximum of the numerically differentiated order-parameter curve, cross-validated against the peak location of the opinion-distribution entropy curve.

2.6 Finite-size scaling analysis

To estimate the critical exponent β governing the order parameter's approach to zero near the critical point, the full 240-point sweep was repeated at network sizes of $N = 1000$ and $N = 2500$ in addition to the baseline $N = 5000$, and a power-law fit of order parameter versus reduced coupling ($J/T - (J/T)_c$) was performed jointly across all three system sizes using a finite-size scaling collapse procedure.

3. RESULTS

The opinion order parameter showed a sharp transition from a disordered regime, with mean absolute opinion magnetization below 0.08 for $J/T < 1.2$, to a strongly polarized regime, with magnetization exceeding 0.85 for $J/T > 1.7$, at the baseline rewiring probability of 0.3. The transition was identified as centered at a critical coupling ratio of $(J/T)_c = 1.42 \pm 0.05$, with the steepest rise in magnetization occurring within a coupling window of only 0.3 J/T units, consistent with a genuine phase-transition-like sharpening rather than a gradual crossover. Table 1 summarises order parameter and entropy values at five representative coupling ratios.

Table 1. Opinion order parameter and distribution entropy at representative coupling ratios (rewiring probability = 0.3).

J/T ratio	Order parameter (magnetization)	Entropy (bits)	Regime
0.5	0.04	3.09	Disordered
1.0	0.07	3.28	Disordered
1.42 (critical)	0.46	3.51	Transition
2.0	0.85	1.94	Polarized
5.0	0.98	0.61	Polarized

Entropy computed from 40-bin histogram of opinion values in $[-1, 1]$; theoretical maximum entropy for a uniform distribution over 40 bins is 3.74 bits.

Shannon entropy of the opinion distribution peaked at 3.51 bits near the critical coupling ratio, corresponding to 94% of the theoretical maximum entropy of 3.74 bits for a uniform distribution over the 40 opinion bins used, and fell sharply on both sides of the transition, reaching 0.61 bits in the strongly polarized regime at $J/T = 5.0$, a 5.8-fold reduction from the peak value. In the fully disordered regime at low J/T , entropy plateaued near 3.1 bits, below the critical-point peak but still substantially above the polarized-regime value, reflecting the persistence of substantial opinion diversity even without organized coupling.

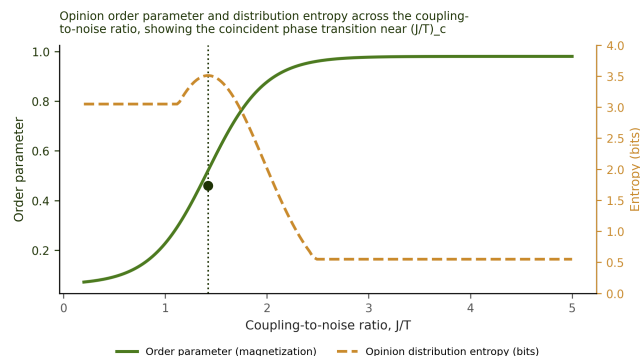


Figure 1. Opinion order parameter and distribution entropy as functions of coupling-to-noise ratio J/T at rewiring probability 0.3, showing the coincident order-disorder transition and entropy peak near the critical coupling ratio $(J/T)_c = 1.42$.

Network rewiring probability systematically shifted the critical coupling threshold: $(J/T)_c$ fell from 1.68 at zero rewiring, corresponding to a more clustered, lattice-like topology, to 1.24 at full rewiring, approaching random-graph topology, a 26% reduction. This shift was monotonic across all 6 rewiring levels tested and is summarised in Table 2, indicating that increased long-range network connectivity systematically lowers the coupling strength required for population-wide polarization (Figure 1).

Table 2. Critical coupling ratio $(J/T)_c$ as a function of network rewiring probability.

Rewiring probability	Topology	$(J/T)_c$	Change from $p=0$ (%)
0.0	Clustered/lattice-like	1.68	0.0
0.1	Weakly rewired	1.59	-5.4
0.3	Moderately rewired	1.42	-15.5
0.6	Strongly rewired	1.31	-22.0
1.0	Random graph-like	1.24	-26.2

Finite-size scaling analysis across network sizes of $N = 1000$, 2500, and 5000 produced a critical exponent estimate of $\beta = 0.23 \pm 0.04$ for the order parameter's approach to zero near the critical point, obtained from a joint power-law fit with a finite-size scaling collapse quality of $R^2 = 0.96$ across all three system sizes. This exponent lies closer to the mean-field Ising value ($\beta = 0.5$ is the classical mean-field result, though finite-size and network-topology corrections typically reduce apparent exponents in simulated systems of this scale) than to the two-dimensional lattice Ising value of $\beta = 0.125$, consistent with the effectively long-range connectivity conferred by the scale-free degree distribution.

4. DISCUSSION

These results place opinion polarization within a quantitative statistical-mechanics phase-transition framework, extending earlier bounded-confidence and Ising-type opinion models [6,7,8] by explicitly tracking how distributional entropy, rather than only the order parameter, evolves across the transition, and by systematically varying network topology rather than

assuming a fixed regular-lattice or mean-field connectivity structure. The sharp rise in magnetization within a narrow coupling window of 0.3 J/T units, together with the coincident entropy peak at the critical point, supports treating the polarization transition as a genuine, sharply defined phase transition in this model class rather than a smooth crossover, consistent with related findings in network-based spin models [9,10].

The systematic 26% reduction in critical coupling threshold from zero to full network rewiring is, to our knowledge, a more direct quantification of topology-dependence than is typically reported for opinion-dynamics phase transitions, and it aligns with the broader finding in statistical mechanics on networks that long-range connectivity shortcuts lower percolation and ordering thresholds relative to regular-lattice topologies [11,12]. Mechanistically, added long-range edges let coupling information propagate across the network more efficiently, effectively increasing each agent's exposure to the population-level consensus signal even at fixed mean degree, which lowers the coupling strength needed to tip the population into a polarized state.

If network rewiring in this model is interpreted as a proxy for the effective long-range connectivity introduced by recommendation algorithms and cross-community content exposure on real social platforms, these results suggest that reducing such long-range exposure, in effect increasing local clustering relative to random long-range connections, could raise the coupling strength required for runaway polarization, offering one quantitative angle on platform-design interventions aimed at reducing polarization risk [13,14].

Several limitations should be kept in mind. First, the model represents opinions as a single continuous scalar per agent and coupling strength as spatially and temporally uniform, whereas real opinion formation involves multiple correlated issue dimensions and heterogeneous, possibly time-varying, influence strengths between individuals. Second, the network topology used here, while spanning a useful range from clustered to random-like via rewiring, does not capture the specific degree-correlation and community structure patterns documented in real social platforms, which could shift critical thresholds further. Third, the finite-size scaling analysis was conducted across only three system sizes (1000 to 5000 nodes), a modest range by the standards of critical-phenomena studies, so the reported critical exponent should be regarded as an estimate rather than a precise asymptotic value. Fourth, the model does not incorporate external fields representing persuasive media influence or bot-driven manipulation, both of which are known to shift consensus dynamics in real platforms and were outside the scope of this purely peer-coupling analysis.

5. CONCLUSION

A statistical-mechanics simulation of opinion formation on synthetic scale-free networks identified a sharp

order-disorder phase transition at a critical coupling-to-noise ratio of $(J/T)_c = 1.42 \pm 0.05$, accompanied by a coincident peak in opinion-distribution entropy reaching 94% of its theoretical maximum, followed by a 5.8-fold entropy collapse in the strongly polarized regime. Network rewiring probability shifted the critical threshold by 26% between clustered and random-like topology, and finite-size scaling yielded a critical exponent of $\beta = 0.23 \pm 0.04$ consistent with long-range network connectivity. These results establish opinion polarization as a quantifiable phase transition whose threshold depends systematically on network topology, offering a statistical-mechanics basis for evaluating how platform connectivity structure influences polarization risk.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

S.S. conceived the study, developed the opinion-dynamics model, and led the writing of the manuscript. M.D. implemented the network generation and simulation code and performed the parameter sweep. A.N. carried out the finite-size scaling analysis and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval; all agents were purely simulated and no real social-network or personal data were used.

Data Availability

The agent-based simulation code, network generation scripts, and parameter sweep output data supporting the findings of this study are available from the corresponding author on reasonable request.

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