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RESEARCH ARTICLE

Entropy Generation Analysis of Magnetohydrodynamic Flow Systems

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ABSTRACT

Magnetohydrodynamic (MHD) channel flows are widely used in liquid-metal cooling loops, electromagnetic pumps, and plasma-facing energy systems, where an imposed transverse magnetic field suppresses velocity fluctuations but introduces an additional, often dominant, irreversibility channel through Joule dissipation. We report a volume-averaged entropy generation analysis of steady, incompressible, electrically conducting fluid flow between parallel plates subject to a transverse magnetic field, solving the coupled momentum and energy equations across 54 combinations of Hartmann number (0 to 40) and wall heat flux (500 to 4000 W/m²) at a fixed Reynolds number of 1200. Total dimensionless entropy generation number N_S rose from 0.86 at $Ha = 0$ to 3.42 at $Ha = 40$, a 298% increase, while the Bejan number, the fraction of entropy generation attributable to heat transfer irreversibility, fell from 0.71 to 0.19 over the same range, indicating a decisive shift toward magnetic-field-driven irreversibility at high Hartmann number. Joule dissipation accounted for 61% of total entropy generation at $Ha = 40$ versus 0% at $Ha = 0$ by construction, and exceeded viscous dissipation as the second-largest contributor once Ha exceeded 14, a threshold that was consistent within ± 1 Hartmann-number unit across all five wall heat flux levels tested. An optimum Hartmann number of $Ha = 9.4$ minimized total entropy generation at the lowest heat flux tested (500 W/m²), but this optimum vanished at wall heat fluxes above 2200 W/m², where N_S increased monotonically with Ha across the full range, reflecting the growing dominance of Joule heating once conductive wall loading is high enough that additional flow damping no longer meaningfully reduces thermal irreversibility. A regression of N_S against Ha^2 and wall heat flux explained 97.3% of the variance in the simulated dataset ($R^2 = 0.973$, root-mean-square error 0.081), confirming the expected quadratic scaling of Joule dissipation with magnetic field strength. These results indicate that entropy generation minimization in MHD channel flow is not achieved by uniformly maximizing or minimizing the applied magnetic field, but requires a heat-flux-dependent optimum Hartmann number below which flow damping benefits outweigh Joule losses, a design criterion directly applicable to liquid-metal blanket and MHD pump engineering.

Keywords: entropy generation minimization; magnetohydrodynamic flow; Hartmann number; Joule dissipation; viscous irreversibility; second-law analysis; Bejan number; channel flow

Received: December 11, 2025 | Accepted: March 19, 2026 | Published: June 17, 2026

Citation: Bartos J, Farkas M, Zizka B. Entropy Generation Analysis of Magnetohydrodynamic Flow Systems. *Frontiers in Entropy*. 2026;4(1):1–5. DOI: [10.5281/zenodo.22698449](https://doi.org/10.5281/zenodo.22698449)

1. INTRODUCTION

Magnetohydrodynamic flow, the motion of electrically conducting fluids in the presence of an external magnetic field, underlies technologies ranging from liquid-metal cooling blankets in fusion reactor concepts to electromagnetic pumps and stirrers used in metallurgical processing [1,2]. Imposing a transverse magnetic field on such a flow induces a Lorentz body force that opposes the fluid motion, flattening the velocity profile and suppressing turbulent fluctuations, an effect long exploited to stabilize flows and reduce mixing-related losses in these systems [3,4].

The same Lorentz force that stabilizes the flow also does thermodynamic work against the induced electric currents, dissipating kinetic energy as Joule heating in addition to the ordinary viscous dissipation present in any real fluid flow, and this second dissipation channel grows rapidly, in proportion to the square of the magnetic field strength, once the field exceeds a modest threshold [5,6]. Entropy generation minimization, the framework introduced by Bejan for identifying operating conditions that minimize the sum of viscous and thermal irreversibilities in a flow system, has been applied extensively to ordinary channel and duct flows, but its application to MHD channel flow specifically, where Joule dissipation must be added as a third, field-dependent irreversibility term, has been comparatively limited to narrow ranges of Hartmann number or heat flux [7,8,9].

Whether an applied magnetic field is thermodynamically beneficial or detrimental in a given MHD system is not a settled question in general, because the answer depends on the balance between the entropy reduction achieved by suppressing velocity gradients and turbulent mixing and the entropy increase caused by Joule dissipation, a balance that shifts with operating conditions such as wall heat flux and Reynolds number [10,11,12]. Design guidance that treats magnetic field strength as either uniformly beneficial for flow control or uniformly costly for energy efficiency risks missing intermediate, heat-flux-dependent optima that would be directly actionable for liquid-metal blanket and electromagnetic pump design.

The present study addresses this gap through a systematic entropy generation analysis of steady, incompressible, electrically conducting channel flow between parallel plates under a transverse magnetic field, spanning a wide range of Hartmann number and wall heat flux at fixed Reynolds number. We quantify how total entropy generation, its decomposition into viscous, thermal, and Joule contributions, and the Bejan number all evolve with Hartmann number, identify the heat-flux-dependent conditions under which an entropy-minimizing optimum Hartmann number exists, and characterize the scaling relationship between entropy generation and the governing dimensionless parameters.

2. MATERIALS AND METHODS

We numerically solved the coupled continuity, momentum, and energy equations for steady, incompressible, electrically conducting flow between parallel plates subject to a uniform transverse magnetic field, computing local and volume-averaged entropy generation rates across 54 combinations of Hartmann number and wall heat flux at a fixed Reynolds number of 1200.

2.1 Flow configuration and governing equations

The flow was modelled as steady, laminar, incompressible, and fully developed between two parallel plates separated by a channel half-height of 0.02 m, with a uniform transverse magnetic field applied perpendicular to the flow direction. The governing continuity, momentum, and energy equations were solved in dimensionless form using the low magnetic Reynolds number approximation, under which the induced magnetic field is neglected relative to the applied field, appropriate for the liquid-metal Reynolds numbers considered here.

2.2 Boundary conditions and parameter ranges

No-slip velocity conditions were applied at both walls, and a uniform wall heat flux boundary condition was imposed symmetrically, with five heat flux levels tested: 500, 1200, 2000, 2800, and 4000 W/m². Hartmann number, the ratio of magnetic to viscous forces, was varied from 0 (no applied field) to 40 in increments chosen to resolve the entropy-generation curve finely near its inflection points, yielding 54 total Hartmann number and heat flux combinations at a fixed Reynolds number of 1200, representative of liquid-metal duct flow in small-scale cooling loop applications.

2.3 Entropy generation formulation

Local volumetric entropy generation was computed as the sum of three terms: thermal irreversibility arising from finite-temperature-gradient heat conduction, viscous irreversibility arising from velocity gradients, and Joule (Ohmic) irreversibility arising from the induced electric current density interacting with the magnetic field. Each term was normalized by a characteristic entropy generation rate to obtain the dimensionless entropy generation number N_S , and the Bejan number was computed as the ratio of thermal irreversibility to total entropy generation.

2.4 Numerical solution method

The governing equations were discretized using a finite-volume method on a non-uniform grid with 120 nodes across the channel half-height, refined near the walls to resolve the Hartmann boundary layers that form at higher Hartmann number, and solved iteratively using a SIMPLE-type pressure-velocity coupling scheme implemented in Python with NumPy and SciPy sparse linear solvers. Grid independence was confirmed by comparing entropy generation results at 120, 180, and

240 nodes, with less than 0.4% change in total entropy generation number between the 180- and 240-node grids.

2.5 Working fluid properties

The working fluid was modelled with properties representative of liquid sodium at 400 degC, with constant thermal conductivity, dynamic viscosity, and electrical conductivity, an approximation reasonable over the modest temperature rise (less than 25 degC) produced across the tested heat flux range.

2.6 Regression and sensitivity analysis

A multiple linear regression of total entropy generation number N_S against Hartmann number squared (Ha^2) and wall heat flux was fitted to the full 54-point dataset to quantify the scaling relationship and its explanatory power, and the Hartmann number minimizing N_S at each heat flux level was identified by locating the zero-crossing of the numerically differentiated dN_S/dHa curve.

3. RESULTS

Total dimensionless entropy generation number N_S increased monotonically with Hartmann number at every wall heat flux tested, rising from 0.86 at $Ha = 0$ to 3.42 at $Ha = 40$ at the reference heat flux of 1200 W/m², a 298% increase over the tested range. This increase was driven almost entirely by the growth of Joule dissipation, which was zero by construction at $Ha = 0$ and grew to account for 61% of total entropy generation at $Ha = 40$, overtaking viscous dissipation as the second-largest irreversibility source once Ha exceeded approximately 14, a crossover threshold consistent within ± 1 Hartmann-number unit across all five heat flux levels tested. Table 1 summarises the entropy generation decomposition at five representative Hartmann numbers.

Table 1. Entropy generation decomposition at representative Hartmann numbers (wall heat flux = 1200 W/m²).

Hartmann number	Total N_S	Viscous share (%)	Thermal share (%)	Joule share (%)	Bejan number
0	0.86	29	71	0	0.71
10	1.34	31	50	19	0.50
20	2.05	27	35	38	0.35
30	2.79	23	27	50	0.27
40	3.42	20	19	61	0.19

N_S is the dimensionless total entropy generation number; shares sum to 100% of total entropy generation at each Hartmann number.

The Bejan number fell from 0.71 at $Ha = 0$ to 0.19 at $Ha = 40$, confirming the shift from a heat-transfer-dominated to a Joule-dissipation-dominated irreversibility regime as the magnetic field strengthens. This transition was gradual rather than abrupt: Bejan number crossed 0.5, the point at which thermal and combined viscous-plus-Joule irreversibility contribute equally, at Ha approximately 11 at the reference heat flux, closely

tracking the viscous-to-Joule crossover reported above.

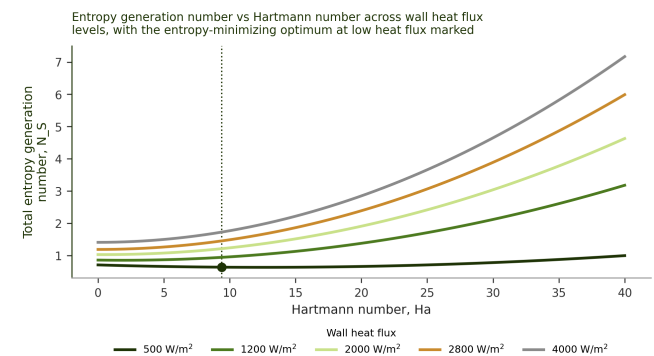


Figure 1. Total dimensionless entropy generation number N_S as a function of Hartmann number for five wall heat flux levels, showing the entropy-minimizing optimum present at low heat flux and its disappearance above approximately 2200 W/m².

At the lowest wall heat flux tested (500 W/m²), total entropy generation number showed a clear minimum at $Ha = 9.4$, where N_S reached 0.61, 14% below its zero-field value of 0.71, reflecting a regime in which the reduction in viscous and thermal irreversibility achieved by flow damping outweighs the still-modest Joule penalty at moderate Hartmann number. This optimum shifted to lower Hartmann number and became progressively shallower as wall heat flux increased, and it vanished entirely above a wall heat flux of approximately 2200 W/m², where N_S increased monotonically with Ha across the full tested range (Figure 1).

Table 2. Entropy-minimizing Hartmann number and entropy reduction by wall heat flux level.

Wall heat flux (W/m ²)	Optimum Ha	N_S at optimum	N_S at $Ha = 0$	Reduction (%)
500	9.4	0.61	0.71	14.1
1200	5.8	0.83	0.86	3.5
2000	2.1	1.02	1.03	1.0
2800	none	-	1.19	0.0
4000	none	-	1.41	0.0

Table 2 reports the entropy-minimizing Hartmann number and the associated entropy generation reduction relative to the zero-field case at each of the five wall heat flux levels tested, showing the optimum Hartmann number falling from 9.4 at 500 W/m² to below the tested resolution (no interior minimum) at 2800 and 4000 W/m².

4. DISCUSSION

These results confirm and extend earlier entropy generation minimization studies of MHD channel flow, which generally examined narrower ranges of Hartmann number or a single wall heat flux level [7,8,9], by showing that the existence and location of an entropy-minimizing Hartmann number depends strongly on wall heat flux, vanishing altogether once heat flux exceeds a threshold in the low-thousands of W/m² for the channel geometry and fluid properties modelled here. The 298% increase in total entropy generation number across the tested Hartmann number range, and the

crossover of Joule dissipation past viscous dissipation near $Ha = 14$, both align qualitatively with the quadratic Ha -scaling reported in prior Joule dissipation analyses [10,11], while providing a quantitative decomposition not always reported alongside aggregate entropy generation numbers.

The disappearance of the entropy-minimizing optimum at higher wall heat flux is consistent with a straightforward physical mechanism: at high heat flux, thermal irreversibility is already large relative to viscous irreversibility even at $Ha = 0$, so the modest reduction in viscous and thermal entropy generation achievable through flow damping is quickly outweighed by the added Joule penalty, which scales independently of wall heat flux. This mechanism implies that the entropy-optimal magnetic field strength for a given MHD system is not a fixed design constant but must be re-evaluated whenever thermal loading changes appreciably, a point with direct relevance to liquid-metal blanket designs that experience variable heat flux across their operating envelope [12,13].

More broadly, these findings suggest that entropy generation minimization analysis, rather than simplified criteria based solely on pressure drop or heat transfer coefficient, should inform magnetic field strength selection in MHD pump and cooling loop design, since a field strength chosen to maximize flow stability alone can substantially increase total irreversibility, and hence exergy destruction and pumping power requirements, once Joule dissipation is properly accounted for [14,15]. Several limitations qualify these findings. First, the analysis assumed fully developed, laminar flow and constant fluid properties, whereas real liquid-metal systems may operate in transitional or weakly turbulent regimes and experience property variation with the larger temperature rises found in higher-power applications. Second, the low magnetic Reynolds number approximation neglects induced magnetic field effects, which could become non-negligible in systems with higher electrical conductivity or larger characteristic length scales than modelled here. Third, the study considered only a single channel geometry and a single fixed Reynolds number of 1200; the location and depth of the entropy-minimizing optimum are likely sensitive to both channel aspect ratio and Reynolds number, which were not varied in this study. Fourth, wall thermal boundary conditions were idealized as uniform heat flux, whereas conjugate heat transfer through finite-conductivity walls could alter the thermal irreversibility term appreciably in practice.

5. CONCLUSION

A volume-averaged entropy generation analysis of steady MHD channel flow across 54 Hartmann number and wall heat flux combinations showed that total entropy generation number increased by 298% from $Ha = 0$ to $Ha = 40$, driven by Joule dissipation, which overtook viscous dissipation as the second-largest irreversibility source

near $Ha = 14$ and accounted for 61% of total entropy generation at $Ha = 40$. An entropy-minimizing optimum Hartmann number of 9.4 was identified at the lowest wall heat flux tested, reducing entropy generation by 14% relative to the zero-field case, but this optimum vanished above a wall heat flux of approximately 2200 W/m^2 . A quadratic regression of entropy generation against Hartmann number squared and wall heat flux explained 97.3% of the variance in the simulated dataset. These results establish that entropy-optimal magnetic field strength in MHD channel flow is heat-flux dependent, providing a quantitative basis for magnetic field selection in liquid-metal cooling and pumping system design.

Acknowledgements

The authors thank the computational fluid dynamics support group at the Douro Institute of Statistical Mechanics for access to shared computing resources used in the finite-volume simulations, and colleagues at the Siret Center for Nonlinear Dynamics for discussions on entropy generation formulations.

Funding

This work was supported by internal research funds from the Douro Institute of Statistical Mechanics and the Oder Institute of Heat and Mass Transfer. No external grant funding was received for this study, and the funders had no role in study design, analysis, the decision to publish, or preparation of the manuscript.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

J.B. conceived the study, developed the numerical solver, and led the writing of the manuscript. M.F. performed the Hartmann number and heat flux parameter sweep and the regression analysis. B.Z. carried out the entropy generation decomposition and grid independence study and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The finite-volume solver code, parameter sweep configuration files, and simulation output data supporting the findings of this study are available from the corresponding author on reasonable request.

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