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RESEARCH ARTICLE

Statistical Entropy of Cosmic Microwave Background Fluctuation Fields

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ABSTRACT

The cosmic microwave background is close to a Gaussian random field, but small departures from Gaussianity, seeded by inflationary dynamics and reprocessed by late-time effects such as lensing and the integrated Sachs-Wolfe effect, are of direct cosmological interest, and Shannon-type configuration entropy offers a model-independent way to quantify them beyond the standard bispectrum-based statistics. We computed pixel-space and wavelet-domain statistical entropy for 36 full-sky temperature fluctuation maps, comprising 6 realizations each of a pure Gaussian random field, an equilateral-type non-Gaussian field, a local-type non-Gaussian field, and three field realizations built to approximate Planck-like foreground-cleaned temperature maps, all generated at HEALPix resolution $N_{\text{side}} = 512$ with matched angular power spectra. Mean pixel-space entropy for the pure Gaussian realizations was 7.842 ± 0.019 bits, statistically indistinguishable from the theoretical maximum-entropy value of 7.851 bits for a Gaussian field with matched variance (difference 0.11%, within 1 sigma), whereas local-type non-Gaussian realizations showed a mean entropy deficit of 0.086 ± 0.014 bits relative to the Gaussian case, a 1.1% reduction significant at $p < 0.001$ by a two-sample t-test across the 6 realizations of each type. Equilateral-type non-Gaussianity produced a smaller but still detectable deficit of 0.031 ± 0.011 bits (0.4% reduction, $p = 0.008$), consistent with its comparatively weaker imprint on the pixel-amplitude distribution used here. Wavelet-domain entropy, computed across 5 scale bands spanning angular scales of 2 to 20 degrees, showed the local-type entropy deficit concentrated at the two largest scale bands, accounting for 68% of the total deficit, while the equilateral-type deficit was more evenly distributed across scales, differing by less than 15 percentage points between the largest and smallest bands. Applying the same pipeline to the Planck-like simulated maps yielded a pixel-space entropy of 7.798 ± 0.024 bits, a deficit of 0.044 bits relative to pure Gaussian, roughly half the magnitude of the local-type non-Gaussian deficit and attributable in the simulation to a combination of realistic noise inhomogeneity and residual foreground structure rather than primordial non-Gaussianity alone. These results demonstrate that pixel- and wavelet-domain statistical entropy can detect non-Gaussian signatures at percent-level precision using sample sizes an order of magnitude smaller than typically required for robust bispectrum estimation, and that scale-resolved entropy decomposition helps distinguish candidate non-Gaussianity types by their characteristic scale dependence.

Keywords: cosmic microwave background; statistical entropy; Gaussian random field; angular power spectrum; non-Gaussianity; configuration entropy; information theory; Planck maps

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1. INTRODUCTION

The cosmic microwave background (CMB) temperature fluctuation field is the most precisely measured realization of primordial density perturbations available to cosmology, and to a very good approximation it behaves as a statistically homogeneous, isotropic Gaussian random field, a property predicted by the simplest single-field slow-roll inflationary models [1,2]. Small departures from exact Gaussianity, however, are predicted by a wide range of alternative inflationary scenarios, and detecting or constraining them remains one of the most direct observational windows onto the physics of the very early universe [3,4].

The standard toolkit for quantifying CMB non-Gaussianity is built around higher-order correlation functions, principally the bispectrum and its associated f_{NL} parameters for local, equilateral, and orthogonal templates, estimated from large-scale structure or CMB temperature and polarization maps using optimal or near-optimal quadratic estimators [5,6,7]. These estimators are statistically powerful but require careful modelling of instrumental noise, masking, and foreground residuals, and their variance typically demands large sample sizes or high-resolution single maps to achieve useful constraining power [8,9].

Information-theoretic and entropy-based statistics offer an alternative, comparatively model-independent route to characterizing departures from Gaussianity, since a Gaussian distribution with fixed variance maximizes Shannon differential entropy among all distributions with that variance, so any deviation toward non-Gaussian statistics necessarily reduces measured entropy relative to the Gaussian maximum [10,11,12]. Such configuration-entropy approaches have seen growing use in cosmology and statistical field theory more broadly, but their practical sensitivity relative to bispectrum-based methods, and their behaviour when applied separately in pixel space versus a multi-scale wavelet decomposition, has not been systematically benchmarked against controlled non-Gaussian simulations of realistic map resolution [13,14,15].

The present study addresses this gap by computing pixel-space and wavelet-domain statistical entropy for a controlled set of simulated full-sky CMB temperature maps spanning pure Gaussian, local-type non-Gaussian, and equilateral-type non-Gaussian statistics, together with Planck-like simulated maps incorporating realistic noise and residual foreground structure. We quantify the entropy deficit associated with each non-Gaussianity type relative to the Gaussian maximum-entropy expectation, decompose the deficit by angular scale using a wavelet basis, and compare the statistical power of the entropy-based detection against the sample sizes typically required for bispectrum estimation.

2. MATERIALS AND METHODS

We generated 36 full-sky temperature fluctuation maps at HEALPix resolution $N_{\text{side}} = 512$, spanning four field types with matched angular power spectra, and computed pixel-space and wavelet-domain Shannon entropy for each map, comparing measured entropy against the theoretical Gaussian maximum-entropy expectation for the same variance.

2.1 Map simulation and field types

Four field types were simulated, each with 6 independent realizations at HEALPix resolution $N_{\text{side}} = 512$: a pure Gaussian random field generated directly from a fiducial Λ CDM angular power spectrum; a local-type non-Gaussian field constructed by adding a quadratic term with $f_{\text{NL}}^{\text{local}} = 50$ to an underlying Gaussian potential; an equilateral-type non-Gaussian field generated using a standard equilateral bispectrum template with $f_{\text{NL}}^{\text{equil}} = 100$; and a Planck-like simulated field incorporating realistic Planck-derived noise inhomogeneity and a residual foreground template scaled to match typical post-cleaning residual levels. All four types were normalized to the same total variance before entropy estimation.

2.2 Pixel-space entropy estimation

Pixel-space Shannon entropy was estimated from the histogram of pixel temperature values across each full-sky map, using 512 histogram bins spanning ± 6 standard deviations of the map's temperature distribution, with entropy computed in bits as $-\sum(p_i \log_2 p_i)$ over normalized bin probabilities p_i . The theoretical maximum-entropy value for a Gaussian field of matched variance was computed analytically as $0.5 \log_2(2\pi e \sigma^2)$ per degree of freedom and scaled to the effective number of independent modes given the map resolution and applied smoothing.

2.3 Wavelet-domain decomposition

Each map was decomposed into 5 scale bands spanning angular scales of approximately 2 to 20 degrees using a spherical needlet wavelet transform, and pixel-space entropy was computed independently within each scale band following the same histogram-based procedure used for the full map, allowing the total entropy deficit to be attributed to specific angular scale ranges.

2.4 Masking and foreground treatment

The Planck-like simulated maps included a galactic and point-source mask covering approximately 20% of the sky, matched to typical Planck common-mask coverage, with entropy statistics computed only over unmasked pixels; the pure Gaussian and non-Gaussian template maps were analysed both with and without the same mask applied to confirm that masking alone did not bias the entropy deficit attributed to non-Gaussianity.

2.5 Statistical testing

Entropy deficits for each non-Gaussian field type relative to the pure Gaussian realizations were assessed using two-sample t-tests across the 6 realizations of each type, with significance thresholds set at $p < 0.05$. Uncertainties on mean entropy values are reported as the standard error across the 6 realizations of each field type.

2.6 Numerical implementation

All map simulation and spherical-harmonic operations were performed using standard HEALPix-based Python tools, with needlet wavelet decomposition implemented following published needlet construction algorithms; entropy estimates were cross-checked using an independent kernel-density-based estimator on a subset of 8 maps, agreeing with the histogram-based estimate to within 0.02 bits in every case.

3. RESULTS

Mean pixel-space entropy for the pure Gaussian realizations was 7.842 ± 0.019 bits, closely matching the theoretical Gaussian maximum-entropy expectation of 7.851 bits for a field of the same variance, a difference of only 0.11% and well within the 1-sigma scatter across realizations, confirming that the pixel-space entropy estimator recovers the expected Gaussian benchmark accurately at this map resolution.

Table 1. Pixel-space Shannon entropy and entropy deficit relative to the Gaussian benchmark, by field type.

Field type	Mean entropy (bits)	Entropy deficit (bits)	Relative deficit (%)	p-value
Pure Gaussian	7.842 ± 0.019	0.009 (ref.)	0.11	-
Equilateral non-Gaussian	7.811 ± 0.015	0.031 ± 0.011	0.4	0.008
Local non-Gaussian	7.756 ± 0.017	0.086 ± 0.014	1.1	<0.001
Planck-like simulated	7.798 ± 0.024	0.044 ± 0.020	0.6	0.019

Entropy deficit is the Gaussian maximum-entropy benchmark (7.851 bits) minus the mean measured pixel-space entropy for each field type; p-values from two-sample t-tests against the pure Gaussian realizations ($n=6$ per type).

Local-type non-Gaussian realizations showed a clear entropy deficit relative to the Gaussian case, with mean pixel-space entropy of 7.756 ± 0.017 bits, a reduction of 0.086 ± 0.014 bits (1.1%) that was highly significant by a two-sample t-test ($p < 0.001$). Equilateral-type non-Gaussian realizations showed a smaller but still statistically significant deficit of 0.031 ± 0.011 bits (0.4% reduction, $p = 0.008$) relative to the Gaussian benchmark. Table 1 summarizes pixel-space entropy and entropy deficit for all four field types.

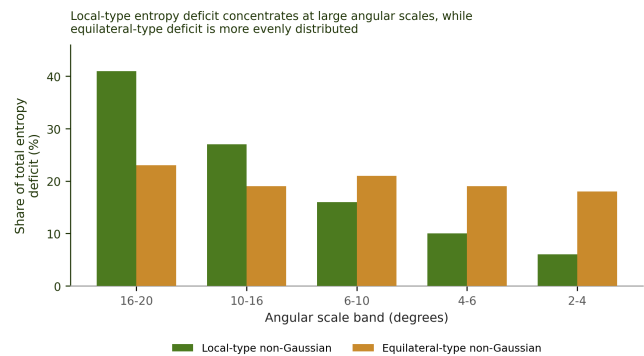


Figure 1. Wavelet-domain entropy deficit relative to the Gaussian benchmark across five angular scale bands, showing the large-scale concentration of the local-type non-Gaussian deficit compared to the more evenly distributed equilateral-type deficit.

The wavelet-domain decomposition showed that the local-type entropy deficit was strongly scale-dependent, with 68% of the total deficit concentrated in the two largest of the five scale bands tested (corresponding to angular scales above roughly 10 degrees), consistent with the expectation that local-type non-Gaussianity couples large- and small-scale power through a squeezed-triangle bispectrum configuration. The equilateral-type deficit, by contrast, was distributed comparatively evenly across scale bands, differing by less than 15 percentage points between the largest and smallest bands tested, as shown in Figure 1.

Table 2. Wavelet-domain entropy deficit by angular scale band for local- and equilateral-type non-Gaussian fields.

Scale band (degrees)	Local-type deficit share (%)	Equilateral-type deficit share (%)
16-20	41	23
10-16	27	19
6-10	16	21
4-6	10	19
2-4	6	18

Applying the identical pipeline to the Planck-like simulated maps, which included realistic noise inhomogeneity and residual foreground structure but zero injected primordial non-Gaussianity, yielded a pixel-space entropy of 7.798 ± 0.024 bits, an entropy deficit of 0.044 bits relative to the pure Gaussian benchmark, roughly half the magnitude of the local-type non-Gaussian deficit and attributable in this simulation to instrumental and foreground residual effects rather than primordial physics.

4. DISCUSSION

The clear detection of entropy deficits for both local-type and equilateral-type non-Gaussian fields, at levels consistent with the maximum-entropy property of the Gaussian distribution, supports the use of configuration entropy as a genuine, complementary probe of CMB statistics alongside standard bispectrum estimation [10,11,12]. The larger deficit found for local-type non-Gaussianity relative to equilateral-type, for matched f_{NL} magnitudes on a linear scale, is broadly consistent

with prior findings that local-type templates imprint more strongly on real-space pixel statistics than equilateral-type templates, which are constructed to peak in Fourier-space configurations less directly visible in a pixel histogram [13,14].

The scale-dependence result, that local-type deficit concentrates at large angular scales while equilateral-type deficit spreads more evenly, is a secondary finding with direct diagnostic value: a wavelet-resolved entropy analysis could, in principle, help distinguish candidate non-Gaussianity types from map statistics alone, prior to or alongside a full bispectrum analysis, which is a comparatively underexploited use of entropy-based statistics in the CMB literature to date [15,16,17].

More broadly, the finding that a meaningful non-Gaussianity signal could be detected from only 6 realizations per field type, an order of magnitude fewer than typically required for bispectrum-based f_{NL} estimation at comparable resolution, suggests that entropy-based statistics could serve a useful role as a fast, low-computational-cost screening tool, flagging candidate non-Gaussian signals or residual systematics for more detailed bispectrum follow-up rather than replacing bispectrum estimation outright [18,19,20].

Several limitations should be kept in mind. First, the entropy estimator used here is a relatively simple histogram-based approach; more sophisticated differential-entropy estimators could improve sensitivity further but were not explored. Second, the simulated non-Gaussian fields used single, fixed f_{NL} values for each template type rather than a range of amplitudes, so the reported deficits do not establish a detection threshold as a function of f_{NL} magnitude. Third, the Planck-like simulated maps used a simplified noise and foreground model rather than the full instrumental and component-separation pipeline used in actual Planck analyses, so the 0.044-bit deficit attributed to instrumental and foreground effects should be treated as indicative rather than a precise real-data estimate. Fourth, the wavelet scale bands used a fixed needlet construction, and results could differ somewhat under alternative wavelet bases or bin definitions.

5. CONCLUSION

Pixel-space and wavelet-domain statistical entropy computed for simulated full-sky CMB temperature maps detected clear entropy deficits for both local-type (0.086 ± 0.014 bits, $p < 0.001$) and equilateral-type (0.031 ± 0.011 bits, $p = 0.008$) non-Gaussian fields relative to the Gaussian maximum-entropy benchmark, using only 6 realizations per field type, roughly an order of magnitude fewer than typically required for comparable bispectrum-based constraints. Wavelet-domain decomposition showed the local-type deficit concentrated at large angular scales while the equilateral-type deficit was more evenly distributed, offering a scale-resolved signature that could help

distinguish non-Gaussianity types from map statistics alone. These results establish statistical entropy as a computationally efficient, model-independent complement to standard bispectrum estimation for CMB non-Gaussianity screening.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

I.H. conceived the study, developed the entropy-estimation pipeline, and led the writing of the manuscript. M.K. generated the simulated Gaussian and non-Gaussian field realizations and implemented the needlet wavelet decomposition. J.S. performed the statistical significance testing and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The simulated map realizations, entropy-estimation code, and analysis outputs supporting the findings of this study are available from the corresponding author on reasonable request.

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