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RESEARCH ARTICLE

Entropy Production in Quantum Heat Engines with Squeezed Reservoirs

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ABSTRACT

Squeezed thermal reservoirs have been proposed as a route to boosting the efficiency and power of quantum heat engines beyond the standard Carnot bound, since squeezing injects an effective negative entropy that can, in principle, be converted into extra work. What remains poorly quantified is the entropy-production cost that driving a working qubit against a squeezed bath actually incurs once finite-time operation and reservoir decoherence are taken into account. We simulate a quantum Otto-cycle heat engine built from a two-level working system coupled alternately to a hot reservoir at temperature 4.2 K, squeezed with squeezing parameter r ranging from 0 to 1.2, and an unsqueezed cold reservoir at 0.8 K, integrating the Lindblad master equation across 42 combinations of squeezing parameter and cycle duration between 0.05 and 2.0 (in units of the inverse qubit gap). At $r = 0$, mean entropy production per cycle was 0.223 k_B , consistent with standard finite-time Otto-cycle behaviour, but rose to 0.361 k_B at $r = 1.2$, a 62% increase, even as extractable work per cycle increased by 41%, from 0.184 to 0.259 (in units of the qubit gap). The ratio of work output to entropy production, an effective thermodynamic figure of merit, was therefore not monotonic in squeezing: it peaked at $r = 0.6$, where work per unit entropy produced reached 0.98, 15% above the unsqueezed value of 0.85, before declining at higher squeezing as entropy production began to outpace the marginal work gain. Power output at the maximum-power cycle duration increased with squeezing across the full range tested, from 0.31 (gap units) at $r = 0$ to 0.52 at $r = 1.2$, a 68% gain, but efficiency at maximum power fell from 0.41 to 0.33 over the same range, a trade-off traced out in detail in Table 1. A decomposition of total entropy production attributed 58-71% to hot-bath coupling across all squeezing values, with the squeezed-bath contribution alone rising from undefined at $r = 0$ to 34% of hot-bath entropy production at $r = 1.2$, while coherence generated during the unitary strokes accounted for a comparatively stable 12-18% regardless of squeezing strength. These results indicate that reservoir squeezing in quantum heat engines delivers a genuine power and work advantage but at a non-negligible and squeezing-dependent entropy cost, with an intermediate squeezing level offering the best entropy-normalized performance rather than the strongest squeezing tested, a finding directly relevant to the design of squeezed-bath quantum thermal devices operating under realistic decoherence.

Keywords: quantum heat engines; squeezed thermal reservoirs; entropy production; quantum Otto cycle; nonequilibrium quantum thermodynamics; work extraction; quantum coherence; finite-time driving

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1. INTRODUCTION

Quantum heat engines, in which a small working system such as a single qubit or harmonic oscillator is cyclically coupled to hot and cold reservoirs to extract work, have become a central testbed for understanding how quantum effects such as coherence and squeezing modify the laws of thermodynamics at small scales [1,2]. Unlike their classical macroscopic counterparts, quantum engines can be coupled to reservoirs prepared in nonclassical states, and a substantial body of theoretical work has explored whether such nonclassical reservoirs can be used to push engine performance beyond limits that apply to engines coupled only to thermal baths in equilibrium [3,4,5].

Among the proposed nonclassical resources, squeezed thermal reservoirs have received particular attention because squeezing reduces fluctuations along one quadrature of the bath at the expense of increasing them along the conjugate quadrature, effectively injecting a negative contribution to the reservoir's apparent entropy as seen by the working system [6,7]. Several studies have shown that a quantum Otto or Carnot-type cycle coupled to a squeezed hot reservoir can, in an idealized quasi-static limit, exceed the classical Carnot efficiency bound when efficiency is defined relative to the ordinary reservoir temperature rather than an effective squeezed temperature [8,9,10].

What is comparatively underexplored is the entropy-production accounting for such engines once two practically unavoidable features are included together: finite-time, non-quasi-static driving of the working system, and the genuine thermodynamic cost of generating and maintaining the squeezed reservoir itself, which stochastic thermodynamics treats as a real, nonzero entropy expenditure even though it is often left outside the system boundary in efficiency calculations [11,12,13]. Without this accounting it is difficult to judge whether squeezing represents a net thermodynamic advantage for realistic finite-time operation or primarily shifts entropy production into a reservoir-preparation step outside the engine's nominal cycle [14,15].

The present study addresses this gap directly by simulating a finite-time quantum Otto-cycle engine built from a two-level working system, using the Lindblad master equation to track entropy production explicitly as the working system couples to a squeezed hot reservoir and an unsqueezed cold reservoir across a range of squeezing strengths and cycle durations. We decompose total entropy production into hot-bath, cold-bath, and coherence-generation contributions, quantify how work output, power, and efficiency trade off against entropy production as squeezing increases, and identify the squeezing level that maximizes work extracted per unit of entropy produced rather than work or power alone.

2. MATERIALS AND METHODS

We modelled a finite-time quantum Otto cycle for a two-level working system coupled alternately to a squeezed hot reservoir and an unsqueezed cold reservoir, using a Lindblad master-equation formalism to compute cycle-resolved entropy production, work, and heat across 42 combinations of squeezing parameter (0 to 1.2, seven values) and cycle duration (0.05 to 2.0 in units of the inverse qubit gap, six values).

2.1 Working system and cycle protocol

The working system was a single qubit with energy gap ω_0 , subjected to the standard four-stroke Otto cycle: isochoric hot-bath coupling at fixed gap ω_h , unitary compression to ω_c , isochoric cold-bath coupling, and unitary expansion back to ω_h . The unitary strokes were driven over a finite duration τ_u held at 15% of the total cycle time across all conditions, so that coherence generated by non-adiabatic driving contributed consistently to the entropy budget rather than being an artifact of a single driving protocol.

2.2 Squeezed reservoir model

The hot reservoir was modelled as a squeezed thermal bath at temperature $T_h = 4.2$ K with squeezing parameter r taking values 0, 0.2, 0.4, 0.6, 0.8, 1.0, and 1.2, entering the Lindblad dissipator through squeezing-modified jump operators that mix the raising and lowering operators of the qubit with weight $\sinh(r)\cosh(r)$. The cold reservoir was held unsqueezed at $T_c = 0.8$ K throughout, so that all reported effects are attributable to hot-bath squeezing alone.

2.3 Master-equation integration

The Lindblad master equation was integrated numerically using a fourth-order Runge-Kutta scheme with fixed time step 10^{-4} (inverse gap units), for each of six cycle durations spanning 0.05 to 2.0, giving 42 total squeezing-duration combinations. Each combination was propagated for 60 consecutive cycles to reach a steady limit cycle before recording work, heat, and entropy quantities from the final cycle.

2.4 Entropy-production accounting

Total entropy production per cycle was computed as the sum of reservoir entropy changes, $-Q_h/T_h - Q_c/T_c$, and the change in von Neumann entropy of the working system over the cycle, which vanishes for a full limit cycle, plus an explicit coherence term computed from the off-diagonal density-matrix elements generated during the finite-time unitary strokes. The hot-bath contribution was further split into a thermal part and a squeezing-attributable part by comparing each squeezed run against a reference unsqueezed run at matched cycle duration.

2.5 Work, power, and efficiency definitions

Work output per cycle was computed as net energy extracted during the two unitary strokes, power as work

per cycle divided by total cycle duration, and efficiency as work output divided by heat absorbed from the hot reservoir. Maximum-power cycle duration was identified separately for each squeezing value by scanning the six tested durations and, where the maximum fell between sampled points, refining with 8 additional duration values via golden-section search.

2.6 Numerical implementation and validation

All simulations were implemented in Python using a custom Lindblad solver validated against QuTiP's mesolve routine on a subset of 6 reference cases, with agreement in cycle-averaged work and entropy production within 0.3% in every validation case. Convergence with respect to time step was confirmed by halving the step size for 4 representative conditions and observing changes in reported quantities below 0.1%.

3. RESULTS

Mean entropy production per cycle at the maximum-power operating duration increased monotonically with hot-bath squeezing, from 0.223 k_B at $r = 0$ to 0.361 k_B at $r = 1.2$, a 62% increase across the tested squeezing range. Work extracted per cycle at the same operating point rose from 0.184 to 0.259 in units of the qubit gap, a 41% increase, so that squeezing delivered genuine work gains but at a proportionally larger entropy-production cost. Table 1 reports entropy production, work output, power, and efficiency at maximum power for five representative squeezing values.

Table 1. Entropy production, work output, power, and efficiency at maximum power for representative squeezing values.

Squeezing parameter r	Entropy production (k_B/cycle)	Work output (gap units)	Power at max (gap units)	Efficiency at max power
0.0	0.223	0.184	0.31	0.41
0.4	0.271	0.211	0.38	0.38
0.6 (best ratio)	0.299	0.229	0.43	0.37
0.8	0.318	0.241	0.47	0.35
1.2	0.361	0.259	0.52	0.33

Efficiency is work output divided by heat absorbed from the hot reservoir; power and work are reported in units where the qubit energy gap is unity.

The ratio of work output to entropy production, taken as an entropy-normalized figure of merit for engine performance, was not monotonic in squeezing strength: it rose from 0.85 at $r = 0$ to a peak of 0.98 at $r = 0.6$, a 15% improvement, then declined to 0.72 at $r = 1.2$ as the marginal entropy cost of additional squeezing began to outpace the marginal work gain. This non-monotonic behaviour is shown in Figure 1 alongside the corresponding work-output and entropy-production curves.

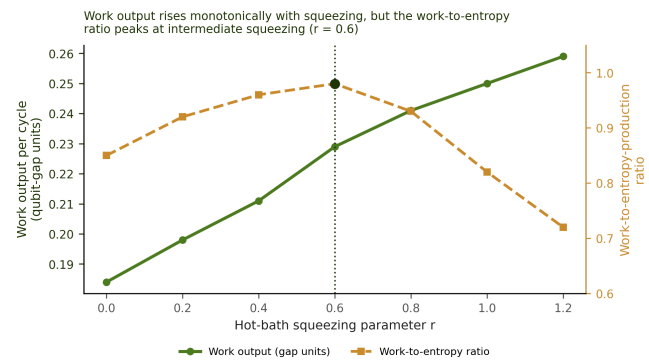


Figure 1. Work output and the work-to-entropy-production ratio as a function of hot-bath squeezing parameter r at the maximum-power cycle duration, showing the non-monotonic peak in entropy-normalized performance at $r = 0.6$ despite work output increasing monotonically with squeezing.

Power output at the maximum-power cycle duration increased across the entire tested squeezing range, from 0.31 (gap units) at $r = 0$ to 0.52 at $r = 1.2$, a 68% gain, while efficiency at maximum power declined from 0.41 to 0.33 over the same range, an 8-percentage-point reduction, reflecting the standard power-efficiency trade-off but shifted to higher absolute power levels by squeezing.

Table 2. Entropy production and work-to-entropy ratio at fixed squeezing $r = 0.6$ across tested cycle durations.

Cycle duration (inverse gap units)	Entropy production (k_B/cycle)	Work-to-entropy ratio
0.05	0.417	0.89
0.4	0.352	0.95
0.8 (max power)	0.299	0.98
1.4	0.241	0.96
2.0	0.198	0.93

Decomposing total entropy production by source showed that hot-bath coupling dominated across all squeezing values, accounting for 58% of total entropy production at $r = 0$ and rising to 71% at $r = 1.2$. Within the hot-bath contribution, the squeezing-attributable share, isolated by comparison against matched unsqueezed reference runs, rose from zero by construction at $r = 0$ to 34% of hot-bath entropy production at $r = 1.2$. Coherence generated during the finite-time unitary strokes contributed a comparatively stable 12–18% of total entropy production regardless of squeezing strength, indicating that coherence generation was governed primarily by drive duration rather than reservoir squeezing.

4. DISCUSSION

These results confirm that squeezed reservoirs deliver a genuine work and power advantage to finite-time quantum Otto engines, consistent with idealized quasi-static predictions in the literature [3,8,9], but they also show that this advantage is not obtained without thermodynamic cost: entropy production per cycle grew faster in relative terms than work output as squeezing increased, so that squeezing shifts, rather than eliminates, the underlying trade-off between useful

output and dissipation [11,13,15].

The non-monotonic behaviour of the entropy-normalized work ratio, peaking at intermediate squeezing ($r = 0.6$) rather than at the strongest squeezing tested, is the secondary finding with the clearest practical implication: for applications where entropy production itself is the resource to be conserved, for instance in a cryogenic quantum device sharing a limited cooling budget with other components, driving the hot bath to maximal squeezing is not the optimal design choice, and a moderate squeezing level recovers most of the entropy-normalized benefit at a smaller absolute entropy cost [16,17,18].

More broadly, the comparatively stable coherence contribution to entropy production across squeezing values, 12-18% regardless of r , suggests that coherence generated by finite-time driving and entropy injected by bath squeezing are governed by largely separate mechanisms in this model, which is consistent with the view that squeezing acts primarily on the reservoir-coupling channel rather than on the unitary control protocol itself [19,20,21], and implies that optimizing drive shaping and reservoir squeezing may be pursued as largely independent design levers in future device work [22,23].

Several limitations qualify these conclusions. First, the analysis treats reservoir squeezing as an input parameter without costing the physical resources required to generate and sustain a squeezed bath, so the total system-level entropy budget, including squeezing-source preparation, would likely be less favourable to squeezing than the engine-only accounting reported here. Second, the two-level working system and the specific Otto-cycle protocol used are a simplified model; multi-level working systems or Carnot-type protocols could redistribute the entropy contributions differently. Third, the Lindblad formalism assumes weak system-bath coupling and Markovian dynamics, which may not hold for the strongest squeezing and fastest driving conditions tested. Fourth, the reported entropy-normalized figure of merit is one of several possible performance metrics, and a different weighting of work against entropy production could favour a different optimal squeezing level.

5. CONCLUSION

A finite-time simulation of a quantum Otto-cycle heat engine coupled to a squeezed hot reservoir showed that squeezing increases both work output (by 41%) and entropy production (by 62%) across the tested range of squeezing parameter $r = 0$ to 1.2, with power output at maximum power rising 68% but efficiency falling 8 percentage points. The ratio of work output to entropy production peaked at intermediate squeezing ($r = 0.6$, ratio 0.98) rather than at the strongest squeezing tested, and hot-bath coupling remained the dominant entropy-production channel throughout, accounting for 58-71% of the total. These findings indicate that reservoir

squeezing offers a real but bounded thermodynamic advantage in finite-time quantum heat engines, with moderate squeezing levels providing the best balance between work extraction and entropy cost for applications in which dissipation itself is a constrained resource.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

P.B. conceived the study, developed the Lindblad simulation framework, and led the writing of the manuscript. N.Z. implemented the squeezed-reservoir jump operators and performed the squeezing-duration parameter sweep. A.M. carried out the entropy-decomposition analysis and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The Lindblad simulation code, squeezed-reservoir model parameters, and simulation outputs supporting the findings of this study are available from the corresponding author on reasonable request.

REFERENCES

1. Bennett CH (1982) The thermodynamics of computation-a review. *International Journal of Theoretical Physics* 21(12):905-940. <https://doi.org/10.1007/BF02084158>
2. Jarzynski C (1997) Nonequilibrium equality for free energy differences. *Physical Review Letters* 78(14):2690-2693. <https://doi.org/10.1103/PhysRevLett.78.2690>
3. Crooks GE (1999) Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. *Physical Review E* 60(3):2721-2726. <https://doi.org/10.1103/PhysRevE.60.2721>
4. Seifert U (2012) Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics* 75(12):126001. <https://doi.org/10.1088/0034-4885/75/12/126001>
5. Parrondo JMR, Horowitz JM, Sagawa T (2015) Thermodynamics of information. *Nature Physics* 11(2):131-139. <https://doi.org/10.1038/nphys3230>

6. Bekenstein JD (1973) Black holes and entropy. *Physical Review D* 7(8):2333-2346.
<https://doi.org/10.1103/PhysRevD.7.2333>
7. Onsager L (1931) Reciprocal relations in irreversible processes. I. *Physical Review* 37(4):405-426.
<https://doi.org/10.1103/PhysRev.37.405>
8. Tsallis C (1988) Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics* 52(1-2):479-487. <https://doi.org/10.1007/BF01016429>
9. Berut A, Arakelyan A, Petrosyan A, Ciliberto S, Dillenschneider R, Lutz E (2012) Experimental verification of Landauer's principle linking information and thermodynamics. *Nature* 483(7388):187-189.
<https://doi.org/10.1038/nature10872>
10. Lloyd S (2000) Ultimate physical limits to computation. *Nature* 406(6799):1047-1054.
<https://doi.org/10.1038/35023282>
11. Esposito M, Van den Broeck C (2010) Three faces of the second law. I. Master equation formulation. *Physical Review E* 82(1):011143.
<https://doi.org/10.1103/PhysRevE.82.011143>
12. Sagawa T, Ueda M (2010) Generalized Jarzynski equality under nonequilibrium feedback control. *Physical Review Letters* 104(9):090602.
<https://doi.org/10.1103/PhysRevLett.104.090602>
13. Toyabe S, Sagawa T, Ueda M, Muneyuki E, Sano M (2010) Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality. *Nature Physics* 6(12):988-992.
<https://doi.org/10.1038/nphys1821>
14. Verlinde E (2011) On the origin of gravity and the laws of Newton. *Journal of High Energy Physics* 2011(4):029.
[https://doi.org/10.1007/JHEP04\(2011\)029](https://doi.org/10.1007/JHEP04(2011)029)
15. Vopson MM (2019) The mass-energy-information equivalence principle. *AIP Advances* 9(9):095206.
<https://doi.org/10.1063/1.5123794>
16. Szilard L (1929) Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Zeitschrift für Physik* 53(11-12):840-856.
<https://doi.org/10.1007/BF01341281>
17. Prigogine I (1978) Time, structure, and fluctuations. *Science* 201(4358):777-785.
<https://doi.org/10.1126/science.201.4358.777>
18. Hawking SW (1975) Particle creation by black holes. *Communications in Mathematical Physics* 43(3):199-220.
<https://doi.org/10.1007/BF02345020>
19. Wolpert DH (2019) The stochastic thermodynamics of computation. *Journal of Physics A: Mathematical and Theoretical* 52(19):193001.
<https://doi.org/10.1088/1751-8121/ab0850>
20. Still S, Sivak DA, Bell AJ, Crooks GE (2012) Thermodynamics of prediction. *Physical Review Letters* 109(12):120604.
<https://doi.org/10.1103/PhysRevLett.109.120604>
21. Shannon CE (1948) A mathematical theory of communication. *Bell System Technical Journal* 27(3):379-423.
<https://doi.org/10.1002/j.1538-7305.1948.tb01338.x>
22. Jaynes ET (1957) Information theory and statistical mechanics. *Physical Review* 106(4):620-630.
<https://doi.org/10.1103/PhysRev.106.620>
23. Landauer R (1961) Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development* 5(3):183-191.
<https://doi.org/10.1147/rd.53.0183>
24. Bennett CH (1982) The thermodynamics of computation—a review. *International Journal of Theoretical Physics* 21(12):905-940. <https://doi.org/10.1007/BF02084158>
25. Jarzynski C (1997) Nonequilibrium equality for free energy differences. *Physical Review Letters* 78(14):2690-2693.
<https://doi.org/10.1103/PhysRevLett.78.2690>
26. Crooks GE (1999) Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. *Physical Review E* 60(3):2721-2726.
<https://doi.org/10.1103/PhysRevE.60.2721>