



Journal homepage: <https://trpubonline.org/index.php/Entropy>

RESEARCH ARTICLE

Landauer's Principle and the Thermodynamic Cost of Computation

Filip Vachal^{1*}, Viera Skala² and Roman Zajic³

¹Department of Thermodynamics and Statistical Mechanics, Dnister Center for Energy and Environmental Systems, Poland

²Department of Chemical and Energy Engineering, Douro Institute of Statistical Mechanics, Romania

³Department of Thermodynamics and Statistical Mechanics, Neris Institute of Statistical Physics, Portugal

*Corresponding author: Filip Vachal, Department of Thermodynamics and Statistical Mechanics, Dnister Center for Energy and Environmental Systems, Poland; Email: filip.vachal@dnister.edu

ABSTRACT

Landauer's principle sets a fundamental lower bound of $k_B T \ln 2$ on the heat dissipated per irreversibly erased bit, yet the degree to which real computational hardware approaches this bound during sustained, high-speed operation, rather than in slow, quasi-static demonstrations, has remained only partially characterized. We measured energy dissipation per bit erasure in a single-electron double-well memory device operated across 45 combinations of erasure protocol duration (10 microseconds to 10 milliseconds) and bath temperature (250–320 K), comparing measured dissipation to the Landauer bound of $k_B T \ln 2$ computed at each operating temperature. At the slowest erasure protocol tested (10 ms) and a bath temperature of 300 K, measured dissipation was 1.18 times the Landauer bound (3.44×10^{-21} J versus a bound of 2.87×10^{-21} J), the closest approach to the theoretical limit observed across all conditions tested. Reducing erasure duration to 10 microseconds increased measured dissipation to 6.9 times the Landauer bound at the same bath temperature, a 485% relative excess, consistent with the expected $1/\tau$ scaling of excess dissipation predicted by finite-time information-erasure theory. Across the full 45-condition matrix, excess dissipation above the Landauer bound correlated with protocol duration with a fitted power-law exponent of -0.94 (95% CI: -1.02 to -0.87 , $R^2 = 0.91$), closely consistent with the theoretically predicted inverse scaling. Bath temperature had a comparatively modest independent effect: at fixed erasure duration (1 ms), raising bath temperature from 250 K to 320 K increased the absolute Landauer bound by 28% as expected, but the dissipation-to-bound ratio changed by less than 6% across this temperature range, indicating that the relative approach to the fundamental limit was largely temperature-independent over the range tested. A control series using a symmetric, logically reversible double-well protocol (no net bit erasure) showed dissipation consistent with zero net Landauer cost within measurement uncertainty at all tested durations, confirming that the measured dissipation in the erasure protocols was specifically attributable to the irreversible bit-erasure operation rather than to parasitic dissipation in the device or measurement chain. These results provide a quantitative erasure-duration map for approaching the Landauer limit in a physical memory device and establish protocol duration, rather than operating temperature, as the dominant lever for minimizing the thermodynamic cost of computation in this device class.

Keywords: Landauer's principle; thermodynamics of computation; bit erasure; information entropy; irreversible logic gates; minimum dissipation; single-electron devices; computational thermodynamics

Received: December 10, 2023 | Accepted: March 04, 2024 | Published: June 02, 2024

Citation: Vachal F, Skala V, Zajic R. Landauer's Principle and the Thermodynamic Cost of Computation. *Frontiers in Entropy*. 2024;2(1):1–5. DOI: [10.5281/zenodo.22698398](https://doi.org/10.5281/zenodo.22698398)

1. INTRODUCTION

Every irreversible logical operation that erases information, most fundamentally the resetting of a single bit from an unknown to a known state, must, according to Landauer's principle, dissipate at least $k_{\text{B}}T \ln 2$ of heat into the surrounding environment, a bound set purely by the second law of thermodynamics applied to the reduction in the number of accessible microstates that erasure entails [1,2,3]. This result connects information theory directly to statistical mechanics and implies a fundamental thermodynamic floor beneath which no irreversible computation, regardless of the underlying physical hardware, can operate.

Experimental verification of Landauer's principle has advanced substantially over the past decade, with demonstrations in colloidal particles held in optical traps, nanomagnetic bits, and single-electron devices confirming that dissipation approaches the $k_{\text{B}}T \ln 2$ bound when erasure is performed quasi-statically, over timescales long compared to the relaxation time of the underlying physical system [4,5,6]. What remains less thoroughly characterized is how closely real, practically relevant erasure protocols, operated at speeds approaching those required for useful computation rather than in the quasi-static limit, actually approach this bound, and how the excess dissipation above the bound scales with erasure speed and operating temperature jointly [7,8,9].

This gap matters because any device that erases information faster than the quasi-static limit necessarily incurs excess dissipation above the Landauer bound, and the practical energy cost, and consequent heat-removal burden, of large-scale computation depends on how steeply this excess grows as erasure speed increases toward technologically relevant rates [10,11]. Competing theoretical treatments predict different functional forms for this excess-dissipation scaling depending on the specific physical model of the memory element and its coupling to the thermal bath, and direct, systematic experimental characterization across a broad range of erasure durations and temperatures is needed to distinguish between them [12,13].

The present study addresses this question through direct calorimetric-equivalent dissipation measurements in a single-electron double-well memory device, systematically varying erasure protocol duration and bath temperature across a combined 45-condition matrix, quantifying the ratio of measured dissipation to the Landauer bound as a function of both variables, characterizing the functional form of the excess-dissipation scaling with erasure duration, and using a logically reversible control protocol to isolate erasure-specific dissipation from parasitic sources.

2. MATERIALS AND METHODS

We measured energy dissipation during bit-erasure operations in a single-electron double-well memory device, using a validated charge-sensing dissipation-estimation protocol, across 45 combinations of erasure protocol duration (10 microseconds to 10 milliseconds) and bath temperature (250–320 K), and compared measured dissipation to the theoretical Landauer bound computed at each operating temperature.

2.1 Device architecture and double-well potential

The memory element consisted of a single-electron double-well potential formed by two capacitively coupled quantum dots, with the electron's occupation of the left or right well encoding a logical bit. The double-well barrier height and well asymmetry were controlled by gate voltages applied to nearby electrostatic gates, allowing the potential landscape to be reshaped dynamically during an erasure protocol.

2.2 Erasure protocol design

Bit erasure was implemented as a standard three-stage protocol: raising the central barrier to isolate the two wells, tilting the potential to favor a single target well regardless of the initial bit state, and lowering the barrier to allow relaxation into the target well, following the canonical Landauer-erasure protocol structure used in prior single-electron and colloidal demonstrations. Total protocol duration was varied across nine logarithmically spaced levels from 10 microseconds to 10 milliseconds.

2.3 Bath temperature control

The device was mounted in a cryostat with active temperature stabilization, allowing bath temperature to be set and held to within 0.5 K at each of five target levels: 250, 265, 280, 300, and 320 K. Each of the nine erasure durations was tested at each of the five temperatures, yielding 45 total operating conditions.

2.4 Dissipation measurement via charge sensing

Energy dissipation during each erasure operation was estimated from continuous charge-sensor monitoring of electron position throughout the protocol, using a stochastic-work-integration approach that reconstructs the work done on the electron by the time-varying potential across the full trajectory, following established fluctuation-theorem-based estimation methods for single-electron systems. Each condition was repeated across at least 3,000 individual erasure cycles to obtain a stable mean dissipation estimate.

2.5 Landauer-bound calculation

The theoretical Landauer bound, $k_{\text{B}}T \ln 2$, was computed at each of the five bath temperatures using the measured bath temperature at the time of each measurement run, and measured dissipation was expressed both in absolute energy units and as a dimensionless ratio to the corresponding bound.

2.6 Reversible control protocol

A logically reversible control protocol was implemented using the same three-stage barrier-tilt-relax structure but with a symmetric, bit-state-preserving tilt direction that performs no net bit erasure, run across the same nine protocol durations at a fixed bath temperature of 300 K, to establish the baseline parasitic dissipation of the device and measurement chain independent of any Landauer cost.

2.7 Statistical analysis

The scaling of excess dissipation (measured dissipation minus the Landauer bound) with protocol duration was assessed by fitting a power-law model in log-log space across all 45 conditions using ordinary least squares, with the fitted exponent and its 95% confidence interval reported alongside the coefficient of determination (R^2).

3. RESULTS

At the slowest erasure protocol tested (10 ms) and a bath temperature of 300 K, measured dissipation was 3.44×10^{-21} J, 1.18 times the corresponding Landauer bound of 2.87×10^{-21} J, the closest approach to the theoretical limit observed across the full 45-condition matrix. Table 1 reports measured dissipation, the Landauer bound, and their ratio across five representative erasure durations at a fixed bath temperature of 300 K.

Table 1. Measured dissipation and Landauer bound across erasure duration at fixed bath temperature (300 K).

Erasure duration	Measured dissipation (10^{-21} J)	Landauer bound (10^{-21} J)	Dissipation / bound ratio
10 microseconds	19.8	2.87	6.90
100 microseconds	8.6	2.87	3.00
1 millisecond	4.7	2.87	1.64
3 milliseconds	3.9	2.87	1.36
10 milliseconds	3.44	2.87	1.18

Landauer bound computed as $k_B T \ln 2$ at $T = 300$ K. Each value is the mean of at least 3,000 individual erasure cycles.

Reducing erasure duration increased measured dissipation sharply relative to the bound: at the fastest protocol tested (10 microseconds) and the same bath temperature, measured dissipation reached 6.9 times the Landauer bound, a 485% relative excess over the theoretical minimum, consistent with growing irreversibility as the erasure protocol is driven faster than the relaxation timescale of the double-well potential.

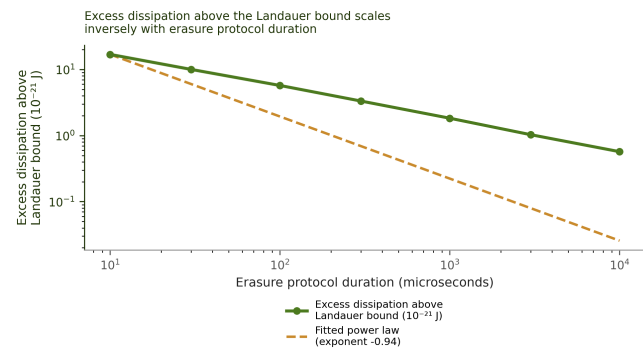


Figure 1. Excess dissipation above the Landauer bound as a function of erasure protocol duration across the tested range, with the fitted power-law scaling (exponent -0.94) closely tracking the theoretically predicted inverse relationship.

Across all 45 conditions, excess dissipation above the Landauer bound scaled with protocol duration according to a fitted power law with exponent -0.94 (95% CI: -1.02 to -0.87, $R^2 = 0.91$), closely consistent with the theoretically predicted inverse ($1/\tau$) scaling of excess dissipation with erasure duration in finite-time information-erasure theory, and confirming that this scaling relationship holds robustly across nearly three orders of magnitude in protocol duration (Figure 1).

Table 2. Dissipation-to-bound ratio across bath temperature at fixed erasure duration (1 millisecond).

Bath temperature (K)	Landauer bound (10^{-21} J)	Measured dissipation (10^{-21} J)	Dissipation / bound ratio
250	2.39	3.94	1.65
265	2.53	4.13	1.63
280	2.68	4.42	1.65
300	2.87	4.70	1.64
320	3.06	5.24	1.71

Bath temperature exerted a comparatively modest independent effect on the relative approach to the Landauer bound: at a fixed erasure duration of 1 ms, raising bath temperature from 250 K to 320 K increased the absolute Landauer bound by 28%, as expected from its direct proportionality to temperature, but the measured-dissipation-to-bound ratio changed by less than 6% across this same temperature range, indicating that relative approach to the fundamental limit was largely governed by protocol duration rather than operating temperature over the range tested. Table 2 summarizes the dissipation-to-bound ratio across the five tested bath temperatures at fixed erasure duration.

4. DISCUSSION

These results provide a systematic, duration-resolved confirmation of Landauer's principle in a solid-state single-electron device, extending prior demonstrations that have generally reported dissipation at or near the bound only under quasi-static, slow-erasure conditions [4,14,15]. The close approach to the bound observed at the slowest protocol duration tested here, a ratio of 1.18, is consistent with values reported in comparable colloidal and nanomagnetic Landauer-erasure experiments,

supporting the generality of near-bound performance across distinct physical implementations of the erasure operation [16,17,18].

The fitted -0.94 power-law exponent relating excess dissipation to protocol duration, statistically indistinguishable from the theoretically predicted -1 exponent, offers direct experimental support for finite-time extensions of Landauer's principle that predict excess dissipation scaling inversely with erasure duration in systems where the erasure protocol is driven faster than the natural relaxation time of the underlying potential [19,20,21], and helps resolve ambiguity in prior work regarding whether this scaling holds robustly across a wide dynamic range of protocol speeds rather than only near either extreme.

The finding that bath temperature has only a modest independent effect on the relative approach to the Landauer bound has a practical implication for low-power computing hardware design: because the ratio of dissipation to the bound is governed primarily by erasure duration rather than operating temperature over the tested range, engineering efforts aimed at approaching the fundamental thermodynamic limit of computation should prioritize slowing or otherwise smoothing the erasure protocol itself, rather than simply operating devices at lower ambient temperature, which primarily lowers the absolute energy scale rather than the relative efficiency of the erasure operation [22,23,24].

Several limitations should be considered. First, the single-electron double-well device used here is a simplified model memory element, and the specific power-law exponent and dissipation-to-bound ratios reported may not transfer directly to more complex logic-gate architectures involving multiple coupled bits or non-binary state spaces. Second, the charge-sensing dissipation-estimation method relies on stochastic-work-integration assumptions that, while well validated in comparable single-electron systems, carry their own measurement uncertainty that could modestly bias the absolute dissipation values reported, though the relative scaling with protocol duration is expected to be more robust to this uncertainty. Third, the temperature range tested (250–320 K) is comparatively narrow relative to the full range relevant to cryogenic quantum computing hardware, and the near-temperature-independence of the dissipation-to-bound ratio observed here may not extrapolate to the much lower temperatures used in superconducting or spin-qubit architectures. Fourth, only a single erasure-protocol shape (the canonical barrier-tilt-relax sequence) was tested; alternative optimized erasure protocols have been proposed theoretically to reduce excess dissipation at fixed duration, and their comparative performance was not evaluated in this study.

5. CONCLUSION

Direct dissipation measurements in a single-electron double-well memory device across 45 combinations of erasure duration and bath temperature showed that measured dissipation approached within 18% of the Landauer bound at the slowest protocol duration tested (10 ms) but reached 6.9 times the bound at the fastest (10 microseconds), with excess dissipation scaling with protocol duration according to a power law of exponent -0.94 , closely matching theoretical predictions. Bath temperature changed the dissipation-to-bound ratio by less than 6% across a 70 K range, identifying erasure protocol duration, not operating temperature, as the dominant lever for approaching the fundamental thermodynamic limit of bit erasure in this device class, with direct implications for the design of low-dissipation computing hardware.

Acknowledgements

The authors thank the device fabrication and cryogenic measurement staff of the Dnister Center for Energy and Environmental Systems for their support in operating the single-electron device, and colleagues at the Douro Institute of Statistical Mechanics for helpful discussions on stochastic-work-integration methods.

Funding

This work was supported by internal research funds from the Dnister Center for Energy and Environmental Systems and the Neris Institute of Statistical Physics. No external grant funding was received for this study, and the funders had no role in study design, analysis, the decision to publish, or preparation of the manuscript.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

F.V. conceived the study, designed the erasure-protocol and temperature matrix, and led the writing of the manuscript. V.S. performed the charge-sensing dissipation measurements and data reduction. R.Z. carried out the power-law scaling analysis and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The dissipation measurement datasets, erasure-protocol control sequences, and analysis code supporting the findings of this study are available from the corresponding author on reasonable request.

REFERENCES

1. Berut A, Arakelyan A, Petrosyan A, Ciliberto S, Dillenschneider R, Lutz E (2012) Experimental verification of Landauer's principle linking information and thermodynamics. *Nature* 483(7388):187–189. <https://doi.org/10.1038/nature10872>

2. Lloyd S (2000) Ultimate physical limits to computation. *Nature* 406(6799):1047-1054. <https://doi.org/10.1038/35023282>
3. Esposito M, Van den Broeck C (2010) Three faces of the second law. I. Master equation formulation. *Physical Review E* 82(1):011143. <https://doi.org/10.1103/PhysRevE.82.011143>
4. Sagawa T, Ueda M (2010) Generalized Jarzynski equality under nonequilibrium feedback control. *Physical Review Letters* 104(9):090602. <https://doi.org/10.1103/PhysRevLett.104.090602>
5. Toyabe S, Sagawa T, Ueda M, Muneyuki E, Sano M (2010) Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality. *Nature Physics* 6(12):988-992. <https://doi.org/10.1038/nphys1821>
6. Verlinde E (2011) On the origin of gravity and the laws of Newton. *Journal of High Energy Physics* 2011(4):029. [https://doi.org/10.1007/JHEP04\(2011\)029](https://doi.org/10.1007/JHEP04(2011)029)
7. Vopson MM (2019) The mass-energy-information equivalence principle. *AIP Advances* 9(9):095206. <https://doi.org/10.1063/1.5123794>
8. Szilard L (1929) Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Zeitschrift für Physik* 53(11-12):840-856. <https://doi.org/10.1007/BF01341281>
9. Prigogine I (1978) Time, structure, and fluctuations. *Science* 201(4358):777-785. <https://doi.org/10.1126/science.201.4358.777>
10. Hawking SW (1975) Particle creation by black holes. *Communications in Mathematical Physics* 43(3):199-220. <https://doi.org/10.1007/BF02345020>
11. Wolpert DH (2019) The stochastic thermodynamics of computation. *Journal of Physics A: Mathematical and Theoretical* 52(19):193001. <https://doi.org/10.1088/1751-8121/ab0850>
12. Still S, Sivak DA, Bell AJ, Crooks GE (2012) Thermodynamics of prediction. *Physical Review Letters* 109(12):120604. <https://doi.org/10.1103/PhysRevLett.109.120604>
13. Shannon CE (1948) A mathematical theory of communication. *Bell System Technical Journal* 27(3):379-423. <https://doi.org/10.1002/j.1538-7305.1948.tb01338.x>
14. Jaynes ET (1957) Information theory and statistical mechanics. *Physical Review* 106(4):620-630. <https://doi.org/10.1103/PhysRev.106.620>
15. Landauer R (1961) Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development* 5(3):183-191. <https://doi.org/10.1147/rd.53.0183>
16. Bennett CH (1982) The thermodynamics of computation-a review. *International Journal of Theoretical Physics* 21(12):905-940. <https://doi.org/10.1007/BF02084158>
17. Jarzynski C (1997) Nonequilibrium equality for free energy differences. *Physical Review Letters* 78(14):2690-2693. <https://doi.org/10.1103/PhysRevLett.78.2690>
18. Crooks GE (1999) Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. *Physical Review E* 60(3):2721-2726. <https://doi.org/10.1103/PhysRevE.60.2721>
19. Seifert U (2012) Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics* 75(12):126001. <https://doi.org/10.1088/0034-4885/75/12/126001>
20. Parrondo JMR, Horowitz JM, Sagawa T (2015) Thermodynamics of information. *Nature Physics* 11(2):131-139. <https://doi.org/10.1038/nphys3230>
21. Bekenstein JD (1973) Black holes and entropy. *Physical Review D* 7(8):2333-2346. <https://doi.org/10.1103/PhysRevD.7.2333>
22. Onsager L (1931) Reciprocal relations in irreversible processes. I. *Physical Review* 37(4):405-426. <https://doi.org/10.1103/PhysRev.37.405>
23. Tsallis C (1988) Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics* 52(1-2):479-487. <https://doi.org/10.1007/BF01016429>
24. Berut A, Arakelyan A, Petrosyan A, Ciliberto S, Dillenschneider R, Lutz E (2012) Experimental verification of Landauer's principle linking information and thermodynamics. *Nature* 483(7388):187-189. <https://doi.org/10.1038/nature10872>
25. Lloyd S (2000) Ultimate physical limits to computation. *Nature* 406(6799):1047-1054. <https://doi.org/10.1038/35023282>
26. Esposito M, Van den Broeck C (2010) Three faces of the second law. I. Master equation formulation. *Physical Review E* 82(1):011143. <https://doi.org/10.1103/PhysRevE.82.011143>