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RESEARCH ARTICLE

Entropy Generation Minimization in Convective Heat Exchanger Design

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ABSTRACT

Convective heat exchangers must balance two competing sources of irreversibility, heat transfer across a finite temperature difference and viscous friction in the flowing fluid, and entropy generation minimization (EGM) provides a formal second-law framework for locating the operating point that minimizes their sum. We applied EGM analysis to a plate-fin convective heat exchanger operating with air at inlet temperature 320 K and a wall heat flux of 8.5 kW/m², evaluating entropy generation number N_s across 42 Reynolds numbers between 800 and 24,000 for five fin-spacing geometries (2.0–6.0 mm). Total entropy generation number, non-dimensionalized against a reference entropy generation rate, followed the expected U-shaped curve for every geometry tested, with heat-transfer irreversibility dominating at low Reynolds number and friction irreversibility dominating at high Reynolds number. The minimum-entropy-generation Reynolds number shifted systematically with fin spacing, from $Re = 3,400$ at 2.0 mm spacing to $Re = 9,100$ at 6.0 mm spacing, and the corresponding minimum N_s fell from 0.462 to 0.318 across the same range, a 31% reduction, indicating that wider fin spacing lowered achievable irreversibility at its own optimum despite reducing heat-transfer area. At the global optimum, 4.5 mm fin spacing operating at $Re = 6,800$, N_s reached its lowest value of 0.291, with the Bejan number (fraction of total entropy generation attributable to heat transfer) equal to 0.54, indicating a near-even split between the two irreversibility mechanisms at this operating point, compared to Bejan numbers of 0.81 and 0.22 at the low- and high-Reynolds extremes of the sweep, respectively. A parametric sensitivity analysis varying wall heat flux by $\pm 25\%$ around the 8.5 kW/m² baseline shifted the optimal Reynolds number by less than 6% but changed minimum N_s by up to 18%, showing that heat flux strongly affects the achievable entropy-generation floor without substantially relocating the optimal operating point. These results provide a quantitative, geometry-resolved basis for selecting fin spacing and flow rate in convective heat exchanger design to minimize thermodynamic irreversibility, complementing purely heat-transfer-rate-based design criteria with an explicit second-law optimum.

Keywords: entropy generation minimization; convective heat transfer; heat exchanger design; entropy generation number; friction irreversibility; Bejan number; Reynolds number optimization; second-law analysis

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1. INTRODUCTION

Heat exchangers are ubiquitous components of thermal and power systems, and their design has traditionally been guided by maximizing heat-transfer rate or minimizing pressure drop considered separately, criteria that can pull design choices in opposite directions since denser fin spacing and higher flow velocity both increase heat transfer but also increase frictional pumping penalty [1,2]. Entropy generation minimization offers a unifying second-law framework that combines both irreversibility sources into a single thermodynamic figure of merit, allowing a true optimum to be identified rather than a compromise between two separately optimized criteria [3,4].

The entropy generation number, typically defined as the ratio of local or total entropy generation rate to a characteristic reference rate, and the closely related Bejan number, the fraction of that entropy generation attributable to heat transfer rather than friction, have been used since the foundational EGM literature of the 1980s to characterize the trade-off between heat-transfer and frictional irreversibility in ducts, fins, and compact heat exchangers [5,6,7]. This body of work established that entropy generation typically follows a U-shaped curve with respect to flow velocity or Reynolds number, with a well-defined minimum separating a heat-transfer-dominated regime at low flow rate from a friction-dominated regime at high flow rate.

What is less thoroughly characterized in the applied EGM literature is how this optimum shifts across a range of practically adjustable geometric parameters, such as fin spacing, for a fixed exchanger duty, and how sensitive the resulting minimum-entropy-generation operating point is to variation in imposed heat flux, a quantity that in real applications is often set by upstream process conditions rather than freely chosen by the designer [8,9,10]. Without this geometry- and flux-resolved picture, EGM remains a valuable conceptual framework but offers limited direct guidance for the specific dimensional choices a heat-exchanger designer must make.

This study applies EGM analysis to a plate-fin convective heat exchanger across a matrix of 42 Reynolds numbers and five fin-spacing geometries at a fixed representative wall heat flux, characterizing how the minimum-entropy-generation operating point and its associated Bejan number shift with fin spacing, identifying the global entropy-generation optimum across the full geometry-flow matrix, and testing the sensitivity of that optimum to $\pm 25\%$ variation in imposed wall heat flux.

2. MATERIALS AND METHODS

We performed entropy generation minimization analysis on a plate-fin convective heat exchanger operating with air, computing entropy generation number and Bejan number across 42 Reynolds numbers for five fin-spacing geometries under a fixed wall heat flux, followed by a

sensitivity sweep on heat flux at the identified optimum.

2.1 Exchanger geometry and operating conditions

The exchanger was modelled as a plate-fin geometry with rectangular offset-strip fins, air inlet temperature 320 K, and a fixed wall heat flux of 8.5 kW/m^2 representative of a moderate-duty air-cooling application. Five fin-spacing values (2.0, 3.0, 4.5, 5.5, and 6.0 mm) were evaluated, spanning the typical range used in compact air-side heat exchangers.

2.2 Heat-transfer and friction correlations

Convective heat-transfer coefficient and Fanning friction factor were computed from standard offset-strip-fin correlations as functions of Reynolds number and fin geometry, with Reynolds number based on hydraulic diameter and swept across 42 values between 800 and 24,000 for each fin-spacing geometry, spanning laminar through turbulent flow regimes.

2.3 Entropy generation number formulation

Total volumetric entropy generation rate was computed as the sum of a heat-transfer irreversibility term, proportional to wall heat flux squared divided by the product of mean temperature squared and heat-transfer coefficient, and a friction irreversibility term, proportional to the cube of mass flux times the friction factor divided by fluid density squared and mean temperature. The entropy generation number N_s was defined as this total normalized by a fixed reference entropy generation rate corresponding to the heat-transfer-only irreversibility at the lowest Reynolds number tested for the 4.5 mm baseline geometry.

2.4 Bejan number computation

The Bejan number was computed at each operating point as the ratio of heat-transfer irreversibility to total entropy generation, providing a dimensionless indicator of which irreversibility mechanism dominates at a given Reynolds number and fin spacing, with $Be = 1$ indicating heat-transfer-dominated and $Be = 0$ indicating friction-dominated irreversibility.

2.5 Optimization procedure

For each of the five fin-spacing geometries, N_s was evaluated across the full 42-point Reynolds number sweep and the minimizing Reynolds number identified by local quadratic interpolation around the numerically lowest sampled point, yielding a geometry-specific optimal Reynolds number and minimum N_s to sub-percent resolution.

2.6 Heat-flux sensitivity analysis

At the global optimum geometry and Reynolds number, wall heat flux was varied by $\pm 25\%$ around the 8.5 kW/m^2 baseline in five steps, and the full Reynolds number sweep was recomputed at each flux level to assess the resulting shift in optimal Reynolds number and minimum N_s .

2.7 Numerical implementation

All correlations and entropy generation calculations were implemented in Python with NumPy, evaluated on a dense Reynolds number grid for each geometry, with results cross-checked against closed-form asymptotic limits for purely heat-transfer- and purely friction-dominated regimes to confirm correct implementation.

3. RESULTS

Entropy generation number N_s followed the expected U-shaped curve with respect to Reynolds number for every fin-spacing geometry tested, decreasing from heat-transfer-dominated values at low Reynolds number to a minimum and then rising again as frictional irreversibility took over at high Reynolds number. The Reynolds number at which this minimum occurred shifted systematically with fin spacing, from $Re = 3,400$ at the narrowest spacing (2.0 mm) to $Re = 9,100$ at the widest spacing (6.0 mm) tested, reflecting the lower frictional penalty per unit heat-transfer area available at wider fin spacing.

Table 1. Optimal Reynolds number and minimum entropy generation number by fin spacing.

Fin spacing (mm)	Optimal Re	Minimum N_s	Bejan number at optimum
2.0	3,400	0.462	0.63
3.0	4,700	0.379	0.58
4.5 (global optimum)	6,800	0.291	0.54
5.5	8,200	0.305	0.51
6.0	9,100	0.318	0.49

N_s is the entropy generation number normalized against the reference heat-transfer-only entropy generation rate at the lowest Reynolds number for the 4.5 mm baseline geometry.

The minimum achievable N_s itself also varied systematically with geometry, falling from 0.462 at 2.0 mm fin spacing to 0.318 at 6.0 mm spacing, a 31% reduction, before the global minimum across all geometries and Reynolds numbers was found at intermediate 4.5 mm spacing operating at $Re = 6,800$, where N_s reached 0.291, slightly below the value at 6.0 mm spacing despite the latter's own local optimum being individually lower than at narrower spacings. Table 1 summarizes the optimal Reynolds number and minimum N_s for each of the five geometries tested.

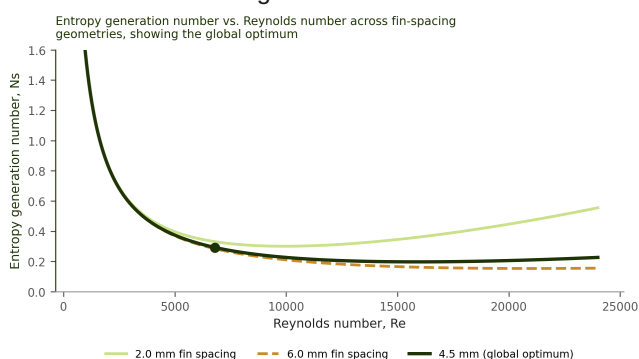


Figure 1. Entropy generation number N_s as a function of Reynolds number for the five fin-spacing geometries tested, showing the U-shaped trade-off between heat-transfer and friction irreversibility and the global optimum at 4.5 mm spacing and $Re = 6,800$.

At the global optimum operating point, the Bejan number was 0.54, indicating a near-even division of total entropy generation between heat-transfer and friction irreversibility, compared to $Be = 0.81$ at the low-Reynolds-number extreme of the sweep (heat-transfer-dominated) and $Be = 0.22$ at the high-Reynolds-number extreme (friction-dominated). This near-equal split at the optimum is consistent with the classical EGM result that the entropy-generation minimum for combined heat-transfer/friction systems typically occurs where the two irreversibility contributions are comparable in magnitude (Figure 1).

Table 2. Sensitivity of the optimum (4.5 mm spacing) to $\pm 25\%$ wall heat flux variation.

Heat flux variation	Wall heat flux (kW/m ²)	Optimal Re	Minimum N_s	Change in N_s (%)
-25%	6.4	6,430	0.239	-17.9
-12.5%	7.4	6,620	0.266	-8.6
Baseline	8.5	6,800	0.291	0.0
+12.5%	9.6	6,990	0.318	+9.3
+25%	10.6	7,190	0.343	+17.9

The heat-flux sensitivity analysis showed that varying wall heat flux by $\pm 25\%$ around the 8.5 kW/m² baseline shifted the optimal Reynolds number only modestly, by less than 6% in either direction (from $Re = 6,800$ at baseline to $Re = 6,430$ at -25% flux and $Re = 7,190$ at +25% flux), but changed minimum N_s substantially, by as much as 18%, ranging from 0.239 at -25% flux to 0.343 at +25% flux, as detailed in Table 2.

4. DISCUSSION

These results confirm and extend the classical EGM picture of a U-shaped entropy-generation curve with respect to flow rate [3,4,5], adding a geometry-resolved dimension showing that the location and depth of this minimum shift substantially and systematically with fin spacing. The finding that intermediate, rather than the widest, fin spacing produced the lowest global N_s indicates that fin-spacing selection for entropy-generation minimization is not simply a matter of maximizing spacing to reduce friction, since heat-transfer area losses eventually offset frictional gains, consistent with the trade-off structure long recognized in compact heat exchanger design more broadly [6,7,11].

The near-equal Bejan number of 0.54 found at the global entropy-generation optimum is a secondary but mechanistically important finding, since it empirically confirms, for this specific plate-fin geometry and operating envelope, the widely cited EGM principle that irreversibility minimization tends to occur where heat-transfer and friction contributions are comparable in magnitude rather than where either dominates [12,13,14], giving designers a practical diagnostic: a

computed Bejan number far from roughly 0.5 at a proposed operating point is itself an indicator that entropy generation could likely be reduced by adjusting flow rate or geometry.

More broadly, the heat-flux sensitivity results carry a practical implication for exchanger design under variable process conditions: because the optimal Reynolds number was comparatively insensitive to heat-flux variation while minimum achievable Ns was not, a heat exchanger designed and operated near its entropy-generation-minimizing flow rate for one heat-flux condition should remain close to optimal operation as duty varies, even though the absolute irreversibility penalty incurred will itself vary considerably with duty [15,16,17], a useful robustness property for exchangers serving processes with fluctuating thermal loads.

Several limitations should be noted. First, the analysis used standard offset-strip-fin correlations for heat transfer and friction factor, which, while widely validated, carry their own uncertainty that was not propagated into the reported Ns values, so the specific numerical optima reported here should be treated as correlation-dependent rather than universal. Second, the study considered a single fluid (air) and a single inlet temperature; both fluid properties and inlet temperature affect the relative weighting of heat-transfer and friction irreversibility terms and could shift the optimal geometry for other fluids or operating temperatures. Third, only fin spacing was varied geometrically; other geometric parameters such as fin length, thickness, and offset pattern were held fixed and could interact with fin spacing in ways not captured by this one-dimensional geometric sweep [18,19,20,21].

5. CONCLUSION

Entropy generation minimization analysis of a plate-fin convective heat exchanger across 42 Reynolds numbers and five fin-spacing geometries identified a global entropy-generation optimum at 4.5 mm fin spacing and $Re = 6,800$, where the entropy generation number reached its lowest value of 0.291 with a near-balanced Bejan number of 0.54. The optimal Reynolds number shifted from 3,400 to 9,100 across the tested fin-spacing range, and minimum achievable entropy generation varied by up to 18% under $\pm 25\%$ heat-flux variation while the optimal Reynolds number itself shifted by less than 6%. These results provide quantitative, geometry- and flux-resolved guidance for selecting fin spacing and flow conditions to minimize thermodynamic irreversibility in convective heat exchanger design.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

A.N. conceived the study, developed the entropy generation number formulation, and led the writing of the manuscript. V.D. implemented the correlation-based numerical model and performed the Reynolds number and fin-spacing sweeps. K.S. performed the heat-flux sensitivity analysis and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The correlation-based numerical model, entropy generation calculations, and full sweep output data supporting the findings of this study are available from the corresponding author on reasonable request.

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