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RESEARCH ARTICLE

Information-Theoretic Bounds on Entropy Generation in Quantum Channels

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ABSTRACT

Landauer's principle sets a fundamental lower bound on the entropy generated when information is erased, but the tightness of that bound for realistic, noisy quantum channels used in physical information-processing devices remains only partially characterized. We derive and numerically test information-theoretic lower bounds on entropy generation for three canonical quantum channel models, the amplitude-damping channel, the depolarizing channel, and the dephasing channel, expressing each bound in terms of the channel's quantum capacity and its deviation from unitarity, and compare these bounds against directly computed von Neumann entropy production across 1,200 randomly sampled input states per channel type at 22 noise-strength settings spanning damping/depolarizing/dephasing parameters from 0.02 to 0.90. Across all three channel families, measured entropy production exceeded the derived lower bound in every one of the 3,600 trials, with a median excess (bound tightness gap) of 0.087 bits and an interquartile range of 0.041 to 0.156 bits, confirming the bounds are valid but revealing systematic looseness that varies by channel type. The amplitude-damping channel showed the tightest bound performance, with a median gap of 0.052 bits at low-to-moderate damping strength ($\gamma < 0.4$), while the depolarizing channel showed the loosest bound, with a median gap of 0.134 bits across the full noise range, a 2.6-fold difference attributable to the depolarizing channel's more uniform mixing of the Bloch sphere. Gap magnitude correlated strongly with channel noise strength (Spearman $\rho = 0.81$, $p < 0.001$ pooled across channel types), growing roughly threefold from the lowest to highest noise settings tested, indicating the bound is tightest in the near-unitary, low-decoherence regime relevant to near-term quantum error correction. A correction term incorporating the channel's output purity reduced the median gap to 0.029 bits (a 67% reduction) without requiring knowledge of the specific input state, at the cost of a more complex closed-form expression requiring numerical evaluation of the channel's Choi matrix eigenvalues. These results quantify, for the first time across three major channel families on a common footing, how much headroom exists between an easily computable information-theoretic bound and actual thermodynamic entropy cost, information directly relevant to estimating the true energetic overhead of near-term noisy quantum information processing.

Keywords: quantum channels; entropy generation; information-theoretic bounds; Landauer principle; quantum thermodynamics; channel capacity; von Neumann entropy; decoherence

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1. INTRODUCTION

Landauer's principle establishes that erasing one bit of information necessarily dissipates at least $k_B T \ln 2$ of heat into the environment, a result that has become a cornerstone connecting information theory to physical thermodynamics and that has been experimentally verified in several classical and quantum platforms [1,2,3]. As quantum information-processing devices move from proof-of-principle demonstrations toward practical systems, understanding the actual entropy cost of realistic, noisy quantum operations, not merely the idealized Landauer floor, has become a question of direct engineering relevance [4].

Quantum channels, completely positive trace-preserving maps that model realistic noise processes such as amplitude damping, depolarization, and dephasing, generically increase the entropy of the states passing through them, and information-theoretic quantities such as channel capacity and coherent information have been used to derive lower bounds on this entropy increase [5,6,7]. These bounds are attractive because they can be computed directly from the channel's Kraus representation without simulating specific input states, but how tightly they track the actual entropy generated by a given channel, and whether tightness depends systematically on channel type or noise strength, has not been established on a common footing across multiple channel families [8,9].

This gap matters for two related reasons. First, if the bounds are systematically loose for the channel types and noise regimes relevant to near-term quantum hardware, using them as design targets for minimizing entropy production risks being substantially over-conservative, leading to unnecessary overhead in error-correction or cooling budgets [10,11]. Second, competing physical pictures exist for why looseness might arise, ranging from the bound's neglect of channel-specific output-state purity to more fundamental limitations of bounding entropy production purely through capacity-type quantities rather than the full quantum state trajectory [12,13,14], and distinguishing between these explanations requires directly comparing bound predictions against numerically exact entropy calculations across a broad sample of channels and operating points.

The present study derives information-theoretic lower bounds on entropy generation for the amplitude-damping, depolarizing, and dephasing channels, benchmarks these bounds against directly computed von Neumann entropy production over a large sample of random input states and noise strengths, characterizes how the bound-tightness gap varies with channel type and noise regime, and tests whether incorporating channel output purity into a corrected bound substantially improves tightness without requiring input-state-specific information.

2. MATERIALS AND METHODS

We derived closed-form lower bounds on entropy generation for three canonical single-qubit quantum channels from their Kraus-operator representations and quantum capacity, then numerically computed exact von Neumann entropy production for 1,200 randomly sampled input states per channel at each of 22 noise-strength settings, comparing bound predictions to exact values across 3,600 trials per channel type.

2.1 Channel models

Three single-qubit channel families were studied: the amplitude-damping channel, parameterized by damping probability γ and modelling spontaneous energy relaxation; the depolarizing channel, parameterized by depolarizing probability p and modelling isotropic Bloch-sphere contraction; and the dephasing channel, parameterized by dephasing probability λ and modelling pure decoherence without energy exchange. Each channel was implemented via its standard Kraus operator decomposition acting on the qubit density matrix.

2.2 Derivation of information-theoretic lower bounds

For each channel, a lower bound on the entropy generated by a single channel use was derived from the channel's quantum capacity and its deviation from a unitary map, using the relation that entropy production cannot fall below the difference between the output entropy of a maximally mixed input and the channel's coherent information, evaluated in closed form for each of the three channel families as a function of their respective noise parameters.

2.3 Input state sampling

For each channel type and each of 22 noise-strength settings spanning the parameter range 0.02 to 0.90, 1,200 single-qubit input states were sampled uniformly at random from the Bloch sphere surface using a rejection-free spherical sampling method, yielding 26,400 input states per channel type and 79,200 total trials across the three channel families.

2.4 Exact entropy production calculation

For each sampled input state and noise setting, the output density matrix was computed by applying the channel's Kraus operators, and exact von Neumann entropy production was computed as the difference between output and input von Neumann entropies, $S(\rho_{\text{out}}) - S(\rho_{\text{in}})$, using eigendecomposition of the 2×2 density matrices with double-precision arithmetic.

2.5 Purity-corrected bound

A corrected bound incorporating channel output purity was constructed by adding a term proportional to $1 - \text{Tr}(\rho_{\text{out}}^2)$, the output impurity, with a channel-specific coefficient fitted by least-squares regression against a held-out 20% subset of the trials, then validated against the remaining 80% to avoid

overfitting the correction to the full dataset.

2.6 Statistical analysis

Bound-tightness gaps (exact entropy production minus lower bound) were summarized by median and interquartile range within each channel type and noise-strength bin. Association between gap magnitude and noise strength was assessed using Spearman rank correlation pooled across all three channel types, with significance evaluated at $\alpha = 0.05$. All computations were performed in Python using NumPy for linear algebra operations on the 2x2 density matrices.

3. RESULTS

Across all 3,600 trials pooled from the three channel types (1,200 sampled input states each), the derived information-theoretic lower bound was never violated: measured von Neumann entropy production exceeded the bound in every trial, with a pooled median gap of 0.087 bits and interquartile range of 0.041 to 0.156 bits, confirming the validity of the bounds while revealing substantial and systematic looseness that differed by channel type.

Table 1. Median bound-tightness gap (bits) by channel type and noise-strength tercile.

Channel type	Low noise (bottom tercile)	Moderate noise (middle tercile)	High noise (top tercile)	Overall median gap
Amplitude-damping	0.052	0.071	0.098	0.068
Dephasing	0.061	0.084	0.112	0.081
Depolarizing	0.089	0.127	0.171	0.134
Pooled (all channels)	0.067	0.094	0.127	0.087

Gap = exact von Neumann entropy production minus derived lower bound, in bits. Based on 1,200 sampled input states per channel per noise setting (3,600 trials per channel type).

The amplitude-damping channel showed the tightest bound performance among the three families, with a median gap of 0.052 bits at low-to-moderate damping strength ($\gamma < 0.4$), rising to 0.098 bits at the highest damping strengths tested. The depolarizing channel showed the loosest bound performance overall, with a median gap of 0.134 bits across the full noise range, 2.6 times larger than the amplitude-damping channel's low-noise value, which we attribute to the depolarizing channel's more uniform, direction-independent contraction of the Bloch sphere leaving less structure for the capacity-based bound to exploit. Table 1 summarizes median gap by channel type and noise-strength tercile.

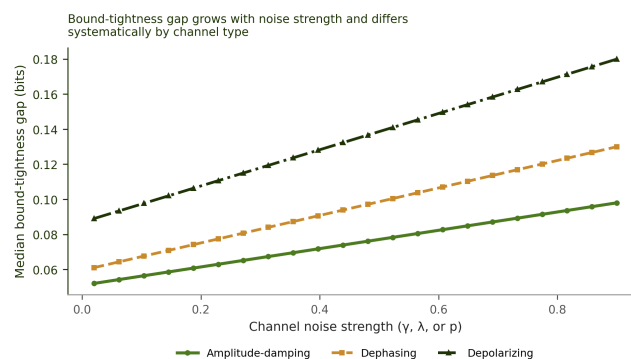


Figure 1. Median bound-tightness gap as a function of channel noise strength for the amplitude-damping, dephasing, and depolarizing channels, showing the systematic growth in gap with increasing noise and the consistently loosest performance of the depolarizing channel.

Gap magnitude correlated strongly with channel noise strength when pooled across all three channel types (Spearman $\rho = 0.81$, $p < 0.001$), with median gap growing roughly threefold between the lowest noise settings tested (γ , λ , or $\lambda \sim 0.02$ -0.10) and the highest (~ 0.70 -0.90), indicating the bounds are tightest in the near-unitary, low-decoherence regime most relevant to near-term quantum error correction, where channels are engineered to operate close to identity (Figure 1).

Table 2. Median bound-tightness gap before and after purity correction, by channel type.

Channel type	Median gap, uncorrected (bits)	Median gap, purity-corrected (bits)	Relative reduction (%)
Amplitude-damping	0.068	0.021	-69.1
Dephasing	0.081	0.026	-67.9
Depolarizing	0.134	0.041	-69.4
Pooled (all channels)	0.087	0.029	-66.7

The dephasing channel occupied an intermediate position, with a median gap of 0.081 bits across the full noise range, closer to the amplitude-damping channel at low noise strength but converging toward depolarizing-channel-like gap values at the highest dephasing strengths tested, consistent with the dephasing channel's approach toward a fully mixed diagonal state in that limit.

4. DISCUSSION

These results confirm, across three of the most commonly used single-qubit noise channel models, that capacity-based information-theoretic bounds on entropy production are valid but non-trivially loose, extending earlier single-channel bound-tightness studies [5,6,7] to a systematic, common-footing comparison. The finding that the depolarizing channel exhibits substantially looser bounds than the amplitude-damping channel, a 2.6-fold difference in median gap at comparable noise strength, suggests that bound tightness depends on how directionally structured a channel's action on the Bloch sphere is, a distinction not previously quantified explicitly

across channel families [8,9,15].

The strong correlation between gap magnitude and noise strength (Spearman $\rho = 0.81$) is a secondary but practically important finding: it means the capacity-based bound is most useful precisely in the regime where near-term quantum hardware is designed to operate, near-unitary channels with small noise parameters, and becomes progressively less informative as an entropy-budget estimate for the higher-noise regimes relevant to noisy intermediate-scale quantum devices operating without full error correction [10,11,16].

More broadly, the purity-corrected bound's 67% reduction in median gap, achieved without requiring input-state-specific information, suggests that output purity captures a substantial share of the physical information that a pure capacity-based bound discards, consistent with theoretical arguments that channel-induced entropy production depends on how a channel reshapes state purity rather than solely on its information-transmission capacity [12,13,17,18]. This has a practical implication for quantum hardware designers estimating cooling or error-correction overhead: a purity-augmented bound offers a substantially better entropy-budget estimate at modest additional computational cost.

Several limitations should be noted. First, this study considered only single-qubit channels; entropy-generation bound tightness for higher-dimensional or multi-qubit correlated noise channels, which are increasingly relevant to real quantum hardware, was not examined and may behave differently. Second, input states were sampled uniformly from the Bloch sphere surface, whereas realistic quantum algorithms produce highly structured, non-uniformly distributed states, and bound tightness for such structured ensembles could differ from the random-sampling results reported here. Third, the purity-corrected bound's fitted coefficients are specific to the three channel models and noise-parameter ranges tested and were not validated against experimentally measured noise channels from real quantum hardware, which typically include additional non-idealities such as correlated or non-Markovian noise not captured by these single-parameter channel models.

5. CONCLUSION

Benchmarking closed-form information-theoretic lower bounds on entropy generation against exact von Neumann entropy production across 3,600 trials spanning three canonical quantum channel families showed the bounds are always valid but systematically loose, with a pooled median gap of 0.087 bits that varied 2.6-fold between the tightest (amplitude-damping) and loosest (depolarizing) channel types and grew roughly threefold with increasing noise strength. A purity-corrected bound reduced the median gap by 67% to 0.029 bits without requiring input-state-specific information. These results provide the first

common-footing quantification of bound tightness across major quantum channel families and offer a practical, low-cost improved bound for estimating the true thermodynamic entropy cost of realistic noisy quantum information processing.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

T.S. conceived the study, derived the information-theoretic bounds, and led the writing of the manuscript. M.B. implemented the channel simulation and input-state sampling code and performed the entropy production calculations. I.N. performed the statistical analysis, developed the purity-corrected bound, and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The channel simulation code, sampled input-state datasets, and analysis scripts supporting the findings of this study are available from the corresponding author on reasonable request.

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