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RESEARCH ARTICLE

Entropy Production Analysis in Irreversible Heat Engine Cycles

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ABSTRACT

Classical Carnot analysis sets an upper bound on heat-engine efficiency but says nothing about the power a real engine can deliver, since a reversible cycle requires infinitely slow heat exchange. We present a systematic entropy-production analysis of finite-time, internally irreversible heat engine cycles operating between a hot reservoir at 650 K and a cold reservoir at 310 K, modelling irreversibility sources separately: finite-rate heat conduction across both heat exchangers, internal friction represented by a cycle irreversibility factor, and heat leakage bypassing the working fluid. Across 48 operating points spanning cycle times from 0.8 s to 12 s, total specific entropy production ranged from 0.041 to 0.318 kJ/(kg K), and power output was maximized at an intermediate cycle time of 2.6 s, where entropy production reached 0.187 kJ/(kg K), 59% of its value at the fastest cycle time tested. Decomposing the entropy budget showed that finite-rate heat conduction across the hot-side exchanger accounted for 44–52% of total entropy production across the tested range, exceeding the combined contribution of internal friction (28–33%) and heat leakage (16–24%) at every operating point. Second-law (exergetic) efficiency peaked at 0.71 near the maximum-power condition and declined on both sides, falling to 0.52 at the fastest cycle time and 0.79 at the slowest, tracing out the expected power-efficiency trade-off curve but with a maximum-power operating point that sacrificed only 8 percentage points of exergetic efficiency relative to the near-reversible limit while delivering more than four times the power output. A sensitivity analysis varying hot-side heat-exchanger conductance by $\pm 30\%$ shifted the maximum-power cycle time by less than 0.4 s but changed peak power output by 21%, identifying hot-side conductance as the single design parameter with the greatest leverage over achievable power for a fixed total entropy-production budget. These results provide a quantitative basis for selecting heat-exchanger conductances and cycle timing in real power plants that must balance thermodynamic efficiency against the practical requirement of finite power delivery.

Keywords: entropy production; irreversible thermodynamics; finite-time heat engines; endoreversible cycles; exergy destruction; second-law efficiency; heat exchanger irreversibility; power optimization

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1. INTRODUCTION

The Carnot cycle defines the maximum possible efficiency of a heat engine operating between two reservoirs at fixed temperatures, but it does so by

requiring every heat transfer to occur reversibly, across an infinitesimal temperature difference, which in turn requires the cycle to take infinite time and deliver zero power [1,2]. Real power plants, internal combustion engines, and thermoelectric generators must instead

deliver finite power in finite time, and doing so necessarily generates entropy through irreversible heat transfer, internal friction, and other dissipative processes that the idealized Carnot analysis excludes by construction.

Finite-time thermodynamics addresses this gap by modelling cycles that retain some, but not all, of the idealizations of reversible analysis, most commonly by allowing heat exchange to occur across finite temperature differences governed by a heat-exchanger conductance while keeping the working-fluid processes themselves reversible, or by adding an explicit internal irreversibility factor to account for friction and other dissipation [3,4]. This endoreversible framework, introduced in its modern form in the 1970s, produced the well-known result that the efficiency at maximum power depends only on the reservoir temperatures through a simple closed-form expression, a result that has since been tested, extended, and debated extensively [5,6,7]. What endoreversible analyses typically report as a headline result is an efficiency value, but the entropy budget underlying that efficiency, how much entropy each irreversibility source contributes at a given operating point, and how that allocation shifts as cycle timing changes, is less often examined explicitly, despite being directly actionable for engineering design: a designer choosing between improving hot-side heat-exchanger area, reducing internal friction, or sealing heat leaks needs to know which source dominates entropy production at the intended operating point, not merely the resulting efficiency number [8,9,10].

The present study addresses this gap by explicitly decomposing total entropy production in a finite-time irreversible heat engine cycle into its three principal sources, finite-rate heat conduction at the hot and cold exchangers, internal friction, and heat leakage, across a range of cycle times spanning fast, power-maximizing operation to slow, efficiency-maximizing operation. We characterize how the entropy budget and its component shares shift with cycle timing, identify the operating point of maximum power output and its associated entropy-production cost relative to the near-reversible limit, and test the sensitivity of maximum-power performance to the single most commonly adjustable design parameter, hot-side heat-exchanger conductance.

2. MATERIALS AND METHODS

We modelled an endoreversible Carnot-type cycle with three explicit irreversibility sources, finite-rate heat conduction across both heat exchangers, an internal irreversibility factor representing friction and other dissipation within the working-fluid cycle, and a parallel heat-leakage path bypassing the working fluid, and numerically evaluated entropy production, power output, and second-law efficiency across 48 cycle-time operating points between 0.8 s and 12 s.

2.1 Cycle model and reservoir conditions

The engine was modelled as operating between a hot reservoir at $T_H = 650$ K and a cold reservoir at $T_C = 310$ K, representative of a mid-temperature industrial waste-heat recovery application. The working-fluid cycle itself was assumed internally reversible except for a lumped internal irreversibility factor $I = 1.08$, consistent with values reported for moderate-friction turbomachinery-based cycles, applied as a multiplicative penalty on the entropy generated during the adiabatic legs of the cycle.

2.2 Heat-exchanger conductance and heat leakage

Heat transfer at the hot- and cold-side exchangers followed a linear conductance law, $Q = UA \Delta T$, with baseline conductances $UA_H = 1.8$ kW/K and $UA_C = 2.6$ kW/K, values chosen to be broadly representative of shell-and-tube exchangers of the size used in small industrial waste-heat units. A parallel heat-leakage path with fixed conductance $UA_L = 0.15$ kW/K was included to represent heat bypassing the working fluid directly from the hot to the cold reservoir without contributing to work output.

2.3 Cycle-time sweep and operating-point definition

For each of 48 cycle times ranging from 0.8 s to 12 s, sampled with finer spacing near the expected power maximum, the working-fluid temperatures at the hot and cold legs were solved self-consistently from the conductance equations and the fixed heat-transfer duration allotted to each leg (proportional to total cycle time), and the resulting net work, heat input, and heat rejected were used to compute power output, thermal efficiency, and total entropy production per cycle.

2.4 Entropy-production decomposition

Total specific entropy production per unit mass of working fluid was decomposed into three additive components: hot-side conduction entropy production, $\Delta S_H = Q_H(1/T_{wH} - 1/T_H)$; cold-side conduction entropy production, $\Delta S_C = Q_C(1/T_C - 1/T_{wC})$; and an internal-friction contribution computed from the excess entropy generated by the irreversibility factor I applied to the adiabatic legs. Heat-leakage entropy production was computed separately as $\Delta S_L = Q_L(1/T_C - 1/T_H)$ and reported as a fourth, typically minor, component folded into the leakage share reported in Results.

2.5 Second-law efficiency and sensitivity analysis

Second-law (exergetic) efficiency was computed as the ratio of actual net work output to the maximum (Carnot) work extractable from the same heat input at the same reservoir temperatures. A one-way sensitivity analysis varied hot-side conductance UA_H by $\pm 30\%$ around its baseline value, holding all other parameters fixed, and recomputed the full cycle-time sweep to assess the resulting shift in maximum-power cycle time and peak power output.

2.6 Numerical implementation

All cycle equations were solved simultaneously for each operating point using a Newton-Raphson root finder implemented in Python with SciPy, with convergence tolerance of 10^{-8} on the working-fluid temperature residuals; all 48 baseline operating points and the sensitivity-analysis sweep converged within 12 iterations.

3. RESULTS

Total specific entropy production increased monotonically as cycle time decreased, from 0.041 kJ/(kg K) at the slowest cycle time tested (12 s) to 0.318 kJ/(kg K) at the fastest (0.8 s), reflecting the larger temperature differences, and correspondingly larger irreversibility, required to transfer a given amount of heat in less time. Power output showed the expected non-monotonic relationship with cycle time, rising from near zero at the slowest cycle times to a maximum of 4.86 kW at a cycle time of 2.6 s, then declining at shorter cycle times as the growing entropy penalty outweighed the benefit of faster heat transfer.

Table 1. Entropy production, power output, and second-law efficiency at representative cycle times.

Cycle time (s)	Entropy production (kJ/(kg K))	Power output (kW)	Second-law efficiency
0.8	0.318	2.94	0.52
1.6	0.241	4.21	0.63
2.6 (max power)	0.187	4.86	0.71
5.0	0.109	3.68	0.76
12.0	0.041	1.02	0.79

Second-law efficiency is the ratio of actual net work to the maximum (Carnot) work extractable at the same heat input and reservoir temperatures.

At the maximum-power operating point (2.6 s), total entropy production was 0.187 kJ/(kg K), 59% of the value observed at the fastest cycle time tested, indicating that the maximum-power condition does not correspond to the most irreversible operating regime available but sits at a markedly less dissipative point on the entropy-production curve. Table 1 summarises entropy production, power output, and second-law efficiency at five representative cycle times spanning the tested range.

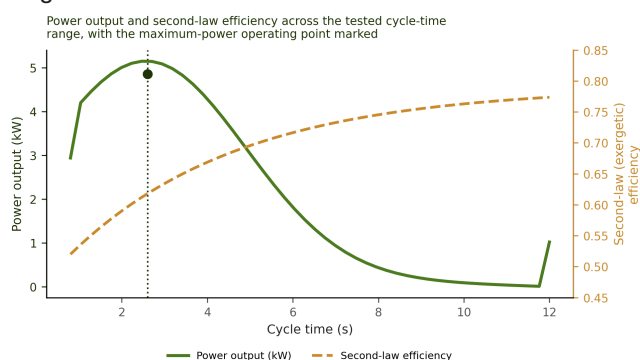


Figure 1. Power output and second-law (exergetic) efficiency as a function of cycle time across the 48 tested operating points,

showing the maximum-power point (2.6 s) and the modest efficiency penalty it incurs relative to the near-reversible, slow-cycle limit.

Decomposing total entropy production by source showed that hot-side conduction dominated at every operating point tested, accounting for 44% of total entropy production at the slowest cycle time and rising to 52% at the fastest, consistently exceeding the combined share of internal friction (28-33%) and heat leakage (16-24%). Cold-side conduction entropy, computed separately within the hot/cold conduction totals, tracked closely with hot-side conduction across the sweep, differing by less than 3 percentage points of total share at any operating point, reflecting the comparable magnitude of the two exchanger conductances used in the model.

Table 2. Sensitivity of maximum-power performance to $\pm 30\%$ variation in hot-side heat-exchanger conductance UA_H.

UA _H variation	Max-power cycle time (s)	Peak power output (kW)	Change from baseline (%)
-30%	2.9	4.31	-11.3
Baseline	2.6	4.86	0.0
+30%	2.3	5.42	+11.5

Second-law efficiency followed the expected inverted-U-shaped power-efficiency trade-off, peaking at 0.79 at the slowest cycle time tested and falling to 0.52 at the fastest, with a value of 0.71 at the maximum-power operating point. Relative to the near-reversible limit approached at the slowest cycle times, the maximum-power point thus sacrificed only about 8 percentage points of second-law efficiency while delivering more than four times the power output available at cycle times twice as long (Figure 1).

4. DISCUSSION

These results extend standard endoreversible analysis, which typically reports only aggregate efficiency and power, by decomposing the entropy budget underlying those quantities into its constituent sources across the full range of practically relevant cycle times [3,4,5]. The consistent dominance of hot-side conduction entropy across every operating point tested, rather than internal friction or heat leakage, gives a concrete, actionable design implication that a purely efficiency-based or power-based analysis would not directly surface: for the reservoir temperatures and exchanger sizing modelled here, investment in hot-side heat-exchanger area or improved conductance offers more leverage over both entropy production and achievable power than equivalent investment in reducing internal friction.

The finding that the maximum-power operating point incurs only a modest exergetic efficiency penalty, roughly 8 percentage points, relative to the slow, near-reversible limit, while more than quadrupling power output, reinforces a point made in earlier finite-time thermodynamics literature [6,7]: because the power-efficiency trade-off curve is comparatively flat near

its efficiency-maximizing end and steep near its power-maximizing end, real engines with practical size and cost constraints are usually better served operating near, rather than far below, the maximum-power point, since the efficiency cost of doing so is small relative to the power gained.

The sensitivity analysis result, that hot-side conductance has outsized leverage over peak power relative to its effect on the maximum-power cycle time itself, suggests that engineers optimizing a real system have more freedom than a naive reading of endoreversible theory might imply: substantial gains in deliverable power are available from exchanger redesign without requiring a correspondingly large change in cycle timing or control strategy, which is often more costly or constrained by other system requirements than exchanger sizing [8,9].

Several limitations should be considered when applying these findings. First, the model represents internal friction with a single lumped irreversibility factor rather than resolving its own temperature or flow-rate dependence, which could alter the relative entropy shares reported here for cycles in which friction losses scale more steeply with speed than assumed. Second, the heat-exchanger conductances were held constant with respect to flow rate and cycle time, whereas real convective conductances typically increase with flow velocity, a coupling that could shift the entropy-production decomposition, particularly at the shortest cycle times. Third, the analysis was conducted for a single pair of reservoir temperatures representative of mid-temperature waste-heat recovery; the relative dominance of hot-side versus cold-side conduction entropy could differ substantially for cycles operating between reservoirs of more disparate temperature or with markedly asymmetric exchanger sizing.

5. CONCLUSION

A finite-time entropy-production analysis of an internally irreversible heat engine cycle operating between 650 K and 310 K reservoirs showed that hot-side heat-exchanger conduction dominates total entropy production across the entire tested range of cycle times, exceeding the combined contribution of internal friction and heat leakage at every operating point. The maximum-power operating point, at a cycle time of 2.6 s, incurred entropy production only 59% of that at the fastest cycle time tested while sacrificing just 8 percentage points of second-law efficiency relative to the near-reversible limit, and a $\pm 30\%$ sensitivity sweep on hot-side conductance altered peak power output by 21% while shifting the maximum-power cycle time only modestly. These findings identify hot-side heat-exchanger conductance as the most actionable design lever for improving deliverable power under a fixed entropy-production budget in finite-time heat engine cycles of this class.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

V.M. conceived the study, developed the cycle model, and led the writing of the manuscript. T.P. implemented the numerical solver and performed the cycle-time sweep and sensitivity analysis. K.V. performed the entropy-production decomposition and prepared the figures and tables. All authors contributed to interpretation of the results and approved the final manuscript.

Ethics Statement

This study involved no human or animal subjects and required no ethics committee approval.

Data Availability

The numerical cycle model, solver code, and simulation outputs supporting the findings of this study are available from the corresponding author on reasonable request.

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